Math 447 **Test 2** **March 17, Spring 2016**

No books, no notes, only SOA-approved calculators. You must show work, unless the question is a true/false or fill-in-the-blank question.

Name: ____

Question	Points	Score
1	6	
2	6	
3	9	
4	9	
5	4	
6	12	
7	6	
8	6	
9	10	
10	5	
Total:	73	

1. Let X has distribution function

$$F(x) = \begin{cases} 0, & y \leq 0, \\ \frac{x}{8}, & 0 < x < 2, \\ \frac{x^2}{16}, & 2 \leq x < 4, \\ 1, & x \geq 4. \end{cases}$$

(a) (3 points) What is the density of X?

(b) (3 points) Find the mean of X.

2. Scores on an examination are assumed to be normally distributed with mean 78 and variance 36.

(a) (3 points) Suppose that students scoring in the top 10% of this distribution are to receive an A grade. What is the minimum score a student must achieve to earn an A grade?

(b) (3 points) If it is known that a student's score exceeds 72, what is the probability that his or her score exceeds 84?

3. Let X have the cdf:

$$F(x) = \begin{cases} 0, & x < -1, \\ \frac{2-x^2}{7}, & -1 \le x < 0, \\ \frac{4}{7}, & 0 \le x < 1, \\ 1, & 1 \le x. \end{cases}$$

(a) (2 points) Find $P(X = 0)$.

(b) (2 points) Let A be the set $\{-2, -1/2, 0, 1, \pi/4\}$. Find $P(X \in A)$.

(c) (2 points) Find $P(-1 \le X \le 0)$.

(d) (3 points) Find $E(X)$.

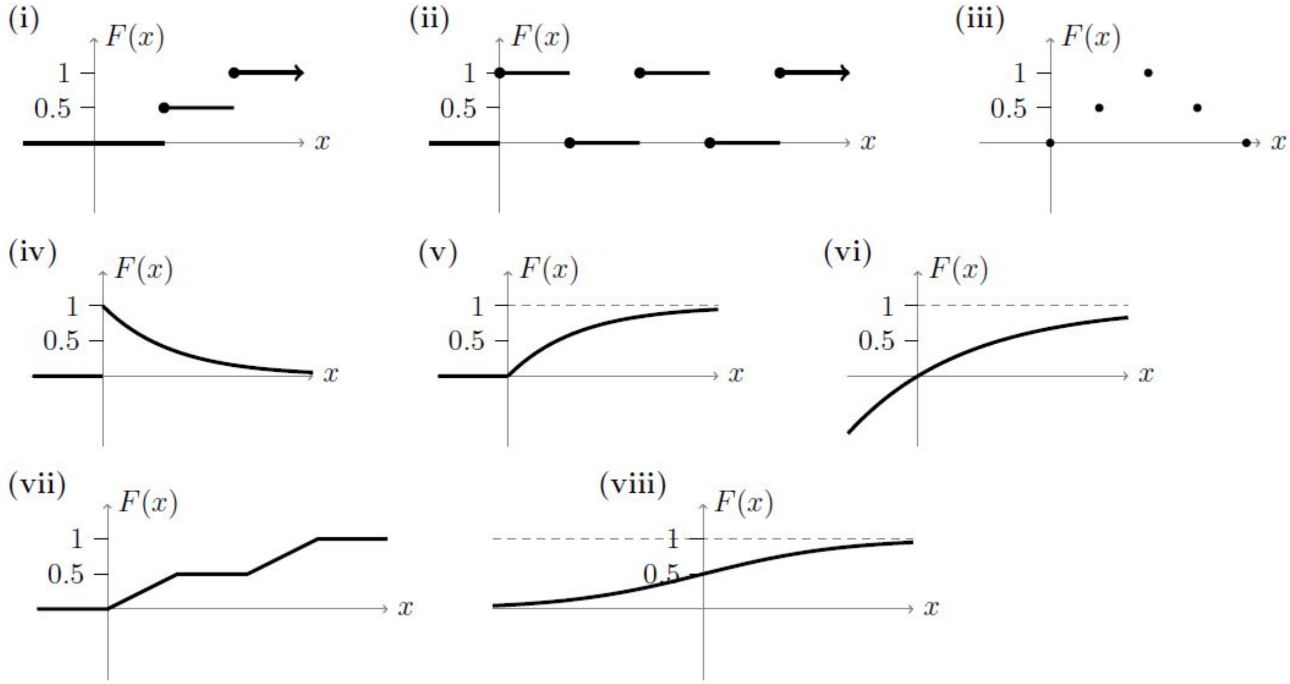
4. A continuous random variable X has pdf $f(x) = x + ax^2$ on $[0, 1]$ and 0 elsewhere.

   (a) (3 points) Find a.

   (b) (2 points) Find the CDF.

   (c) (2 points) Find $P(.5 < X < 1)$.

   (d) (2 points) Find the mean of X.



5.

[No work needed for this question.]

(a) (2 points) Which functions above are valid cdf's?

(b) (2 points) Which functions are cdf's of continuous r.v.'s?

6. Let X be an exponential r.v. with parameter $\beta = 1$.

   (a) (2 points) Find the median of this distribution.

   (b) (3 points) Find $P(|X - \mu| \geq 2\sigma)$, where μ and σ are the expectation and standard deviation of X.

   (c) (2 points) What is the estimate for this probability given by the Chebyshev theorem?

   (d) (3 points) What is $P(X^2 + 2 \geq 3X)$?

   (e) (2 points) What is $E(X^7)$?

7. Let X be an random variable with moment generating function $(1-2t)^{-2}$. Let Y be an random variable with moment generating function $1/(1-3.2t)$. Let Z be an random variable with moment generating function e^{5t+6t^2}.

[No work needed.]

(a) (2 points) What is the distribution of X?

(A) normal
(B) beta
(C) uniform
(D) chi-square
(E) exponential
(F) none of the above

(b) (2 points) What is the distribution of Y?

(A) uniform
(B) chi-square
(C) beta
(D) exponential
(E) normal
(F) none of the above

(c) (2 points) What is the distribution of Z?

(A) chi-square
(B) beta
(C) gamma
(D) uniform
(E) normal
(F) none of the above

8. The minimum force required to break a particular type of cable is normally distributed with mean $12,432$ and standard deviation 25. A random sample of 400 cables of this type is selected.

   (a) (3 points) What is the probability that a randomly chosen cable will break under a force of $12,400$?

   (A) 2.5%
   (B) 5%
   (C) 10%
   (D) 20%
   (E) Other

   (b) (3 points) Calculate the probability that at least 349 of the selected cables will not break under a force of $12,400$.

   [Don't forget to show your work.]

   (A) 0.62
   (B) 0.67
   (C) 0.92
   (D) 0.97
   (E) 1.00

9. A man possesses five coins, two of which are double-headed, one is double-tailed, and two are normal. He shuts his eyes, picks a coin at random, and tosses it.

   (a) (2 points) What is the probability that the lower face of the coin is a head?

   (b) (2 points) He opens his eyes and sees that the coin is showing heads; what is the probability that the lower face is a head?

   (c) (2 points) He shuts his eyes again, and tosses the coin again. What is the probability that the lower face is a head?

   (d) (2 points) He opens his eyes and sees that the coin is showing heads; what is the probability that the lower face is a head?

   (e) (2 points) He discards this coin, picks another at random, and tosses it. What is the probability that it shows heads?

10. (5 points) An insurance policy covers losses incurred by a policyholder, subject to a deductible of $10,000$. Incurred losses follow a normal distribution with mean $12,000$ and standard deviation c. The probability that a loss is less than k is 0.9582, where k is a constant. Given that the loss exceeds the deductible, there is a probability of 0.9500 that it is less than k.

Calculate c.

(A) 2045

(B) 2267

(C) 2393

(D) 2505

(E) 2840