Math 447 **Test 1** **February 25, Spring 2016**

No books, no notes, only scientific (non-graphic) calculators. You must show work, unless the question is a true/false or fill-in-the-blank question.

Name: ____

Question	Points	Score
1	9	
2	6	
3	8	
4	6	
5	3	
6	3	
7	6	
8	6	
9	6	
10	7	
Total:	60	

1. A friend who works in a big city owns two cars, one small and one large. Three-quarters of the time he drives the small car to work, and one-quarter of the time he drives the large car. If he takes the small car, he usually has little trouble parking, and so is at work on time with probability 0.9. If he takes the large car, he is at work on time with probability 0.6. Given that he was on time on a particular morning, what is the probability that he drove the small car?

   Solve this problem according to the following scheme. Make sure to give an appropriate answer for each part.

   (a) (3 points) Establish notation: name the relevant events.

   Let S = event that friend drives small car
   and L = event that friend drives large car
   and T = event that friend is at work on time

   (b) (3 points) Write down the equation that lets you compute the answer, in the notation of part (a).

   $$P(S|T) = \frac{P(T|S)P(S)}{P(T|S)P(S) + P(T|L)P(L)}$$

   (c) (3 points) Solve the problem, giving your answer as a percentage.
   Given: $P(S) = 0.75$, $P(L) = 0.25$, $P(T|S) = 0.90$, $P(T|L) = 0.60$.

   $$P(S|T) = \frac{0.9 \cdot 0.75}{0.9 \cdot 0.75 + 0.6 \cdot 0.25} = \frac{0.675}{0.825} = 0.818 = 81.8\%$$

2. If two events, A and B, are such that $P(A) = 0.5$, $P(B) = 0.3$, and $P(A \cap B) = 0.1$ find the following:

(a) (2 points) $P(A|B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{3}$$

(b) (2 points) $P(B|A)$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1}{5}$$

(c) (2 points) $P(A|A \cup B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.7$$

$$P(A|A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)} = \frac{P(A)}{P(A \cup B)} = \frac{5}{7}$$

3. In southern California, a growing number of individuals pursuing teaching credentials are choosing paid internships over traditional student teaching programs. A group of eight candidates for three local teaching positions consisted of five who had enrolled in paid internships and three who enrolled in traditional student teaching programs. All eight candidates appear to be equally qualified, so three are randomly selected to fill the open positions. Let Y be the number of internship-trained candidates who are hired.

(a) (2 points) Does Y have a binomial or hypergeometric distribution?

Hypergeometric

(b) (2 points) Find the probability that two internship trained candidates are hired.

$$\frac{\binom{5}{2}\binom{3}{1}}{\binom{8}{3}} = \frac{10 \cdot 3}{56} = 53.6\%$$

(c) (2 points) What is the mean of Y ?

$$n\frac{r}{N} = 3\frac{5}{8} = 1.875$$

(d) (2 points) What is the variance of Y ?

$$n\frac{r}{N}(1 - \frac{r}{N})\frac{N-n}{N-1} = 3\frac{5}{8}\frac{3}{8}\frac{5}{7} = 0.502$$

4. **For this question you may leave the answers in terms of binomial coefficients or factorials.**

A standard deck of cards has 52 cards, 4 of each type (Ace, King, Queen, Jack, 10,...,2). From a well shuffled deck, you are dealt a hand of 5 cards (without replacement).

(a) (3 points) What is the probability that you are dealt at least one face card (that is a king, queen or jack)?

There are 12 face cards so

$$P[\text{No Face card}] = \frac{\binom{40}{5}\binom{12}{0}}{\binom{52}{5}} = \frac{\binom{40}{5}}{\binom{52}{5}}$$

Hence, the probability of at least one face card is

$$1 - \frac{\binom{40}{5}}{\binom{52}{5}} = 74.7\%$$

(b) (3 points) What is the probability that you are dealt both at least one ace and at least one face card?

$$P[\text{No Ace or Face card}] = \frac{\binom{36}{5}\binom{16}{0}}{\binom{52}{5}} = \frac{\binom{36}{5}}{\binom{52}{5}}$$

So the probability of at least one ace or one face card is

$$1 - \frac{\binom{36}{5}}{\binom{52}{5}}$$

Then

$$P[\text{An Ace and a Face card}] = P[\text{An Ace}] + P[\text{A Face card}] - P[\text{An Ace or a Face card}]$$

$$= 1 - \frac{\binom{48}{5}}{\binom{52}{5}} + (1 - \frac{\binom{40}{5}}{\binom{52}{5}}) - (1 - \frac{\binom{36}{5}}{\binom{52}{5}}) = 23.3\%.$$

5. (3 points) Three cards are randomly selected, without replacement, from an ordinary deck of 52 playing cards. Compute the conditional probability that the first card selected is a spade, given that the second and third cards are spades.

Let S be the event that the first card is a spade, SSS be the event that the first three cards are spades, and *SS be the event that the two second cards are spades.

Then

$$P(S| * SS) = \frac{P(SSS)}{P(*SS)} = \frac{P(SSS)}{P(SSS) + P(\bar{S}SS)} = \frac{\frac{13}{52}\frac{12}{51}\frac{11}{50}}{\frac{13}{52}\frac{12}{51}\frac{11}{50} + \frac{39}{52}\frac{13}{51}\frac{12}{50}}$$

,

$$P(S| * SS) = \frac{1}{1 + \frac{39}{11}} = \frac{11}{50} = 22\%$$

6. (3 points) If the number of phone calls to the fire department, Y, in a day has a Poisson distribution with mean 7.2, what is the most likely number of phone calls to the fire department on any day? Why?

   It equals 7, because the probability of $Y = y$ increases from $y = 0$ to $y = 7$ and decreases thereafter.

7. The number of misprints on a page has a Poisson distribution with parameter $\lambda = 1$, and the numbers on different pages are independent.

   (a) (3 points) What is the probability that the second misprint will occur on page 1?

   Let X be the number of the misprints on the first page. Then the required probability is

$$P(X \geq 2) = 1 - P(X \leq 1) = 1 - e^{-1} - e^{-1} = 26.4\%$$

   (b) (3 points) What is the probability that the second misprint will occur on page 2?

   Let X and Y be the number of misprints on the first and second pages, respectively. Then the required probability is

$$P(X = 0, Y \geq 2) + P(X = 1, Y \geq 1) = P(X = 0)(Y \geq 2) + P(X = 1)P(Y \geq 1)$$
$$= e^{-1}(1 - 2e^{-1}) + e^{-1}(1 - e^{-1}) = e^{-1}(2 - 3e^{-1}) = 33\%$$

8. Suppose that there is a 1 in 50 chance of injury on a single skydiving attempt.

   (a) (3 points) If we assume that the outcomes of different jumps are independent, what is the probability that a skydiver is injured if she jumps twice?

$$1-(1-\frac{1}{50})^2 = 2\frac{1}{50} - (\frac{1}{50})^2 = 3.96\%$$

   (b) (3 points) What is the probability of suffering at least one injury in 50 jumps.

$$1-(1-\frac{1}{50})^{50} = 63.6\%$$

9. In a class of 20 students each student tosses a fair coin 4 times independently.

   (a) (3 points) What is the probability that no student gets 4 heads. Probability to get 4 heads is $(1/2)^4 = 1/16$.
   Probability that no student gets 4 heads is

$$(1-\frac{1}{16})^{20} = 27.5\%$$

   (b) (3 points) What is the expected number of students who get 4 heads.

$$20\frac{1}{16} = 1.25$$

10. A student is taking a multiple choice test with 10 questions for which she has mastered 60% of the material. Assume this means that she has a 0.6 chance of knowing the answer to a random test question, and that if she does not know the answer to a question then she randomly selects among the four answer choices. Finally, assume that this holds for each question, independent of the others.

(a) (2 points) What is the probability that the student correctly answers question 1?

The probability the the student gets the question correct is

$$P[\text{Correct}] = P[\text{Correct}|\text{Certain of answer}]P[\text{Certain of answer}]$$

$$+P[\text{Correct}|\text{Guesses randomly}]P[\text{Guesses randomly}]$$

$$= 1 \cdot 0.6 + \frac{1}{4} \cdot 0.4 = 0.7$$

Let p be the probability of getting a random question correct. You likely found p in part (a), but in any case you should assume $0.7 \leq p \leq 0.9$.

In parts (b) and (c) you can just use the letter p for this probability.

(b) (3 points) What is the probability that the student was certain of the answer to question 1 given that they got it correct. (Do not simplify the expression you get.)

$$P[\text{Certain of answer}|\text{Correct}] = \frac{P[\text{Certain of answer, Correct}]}{P[\text{Correct}]} = \frac{0.6}{p} = \frac{0.6}{0.7} = 85.7\%$$

(c) (2 points) What is the probability that she will answer at least 9 questions correctly? The number of correct solutions X is $\text{Bin}(10, 0.7)$. Hence

$$P[X \geq 9] = P[X = 9] + P[X = 10] = 10 \cdot (0.7)^9 \cdot 0.3 + (0.7)^{10} = 14.9\%$$