Math 447 Spring 2015 Exam 3 April 21, 2015

- Total value 220 points. Each part valued as indicated.
- SHOW YOUR WORK unless otherwise indicated. "NO WORK" may result in "NO POINTS".
- Simplify your answers when possible. Do the arithmetic, remove parentheses, reduce fraction, etc.
- Cross out anything you don't want graded!
- Use the back sides of pages if you need extra space. If you have anything on a back side that you want graded, indicate where it is.

Student: _____

Problem #	Possible Points	Points
Ι	30	
II	14	
III	21	
IV	36	
V	30	
VI	28	
VII	28	
VIII	33	
Total	220	

Section (please circle): 01 (Xu)

I. (30 points.) X has probability mass function (PMF; i.e., $p(x) = P\{X = x\}$) given in the table below

(1) (8 points) Give the distribution function (CDF) of X. I.e., $F(x) = P\{X \le x\} = ?$

(2) (6 points) $EX^4 = ?$

(3) (6 points) Var X = ?

(4) (6 points) Define $Y = \frac{1}{X+1}$. Give the moment generating function (MGF) of Y, $m_Y(t) = ?$.

(5) (4 points)Find the *q*th quantile of X with $q = \frac{1}{4}$. (No work need be shown.)

II. (14 points.) $\{X_1, \ldots, X_n\}$ is an independent collection of random variables having finite mean μ and finite variance σ^2 . Let $S = \sum_{k=1}^{3} k X_k$. Your answers to the questions below will involve μ and σ . (1) (4 points.) E(S) = ?

(2) (4 points) Var(S) = ?

(3) (6 points) If $Y = X_1 + 2X_2 - 3$ and $Z = X_1 - 2X_2 + X_3 + 4$, then Cov(Y, Z) = ?

III. (21 points.) X has probability density function (PDF)

$$f_X(x) = \begin{cases} \frac{3x^2}{8} & 0 < x < 2\\ 0 & \text{otherwise.} \end{cases}$$

(1) (6 points) Give the distribution function (CDF) of X. I.e., $F(x) = P\{X \le x\} = ?$ (Be sure to give all pieces of this function. Otherwise you will lose points.)

(2) (6 points) EX = ?

(3) (3 points) $P\{X = 1/3\} = ?$

(4) (6 points)
$$P\{X^2 < \frac{1}{4}\} = ?$$

IV. (36 points.) X has distribution function (CDF)

$$F_X(x) = \begin{cases} 0 & x < 0\\ \frac{1+x^2}{10} & 0 \le x < 2\\ 1 & 2 \le x \end{cases}$$

(1) (5 points) Does this distribution function (CDF) have any jumps? If yes, identify all the jump points and the associated probabilities (i.e. P(X = x) =?).

(2) (5 points)
$$P\left\{-\pi < X \le \frac{1}{2}\right\} = ?$$

(3) (5 points) $P\{0 \le x \le 2\} = ?$

(4) (3 points) A is the set
$$\left\{-1, 0, \frac{1}{\sqrt{2}}, 2\right\}$$
. $P\{X \notin A\} = ?$

(5) (2 points) Find F'(x) = ?

(6) (8 points) EX = ?

(7) (8 points) Var(X) = ? (give a summation of reduced fractions as your final answer, no need to simplify.)

V. (30 points.) X is normally distributed with mean $\mu = 3$ and variance $\sigma^2 = 36$. (1) (5 points) $P\{3 \le X \le 15\} = ?$

(2) (5 points) $P\{|X-6| \le 9\} = ?$

(3) (5 points) Find out a value for x_0 so that $P\{X \ge x_0\} = 0.99$

(4) (5 points) $P\{X^3 + 27 \ge 0\} = ?$

(5) (5 points) Give the moment generating function (MGF) for $X, m_X(t) = ?$

(6) (5 points) Let Y = 5X + 10. Give the probability density function (PDF) for Y. I.e., $f_Y(x) = ?$

VI. (28 points.) X has a cumulative distribution function (CDF). (If you don't pay attention to the difference between CDF and PDF, you can say goodbye to the following 33 pts)

$$F_X(x) = \begin{cases} 1 - e^{-5x} & x > 0\\ 0 & x \le 0 \end{cases}$$

- (1) (4 points) E(5X + 1) = ?
- (2) (4 points) Var(5X + 1) = ?
- (3) (5 points) Give the moment generating function (MGF) of X, $m_X(t) = ?$
- (4) (5 points) $E[(5X)^{20}] = ?$
- (5) (5 points) $P\{X \le \frac{1}{10} \mid X \ge \frac{1}{20}\} = ?$

(6) (5 points) Let Y = 10X. Give the distribution function (CDF) of Y. I.e., $F_Y(y) =$? Moreover, based on $F_Y(y)$, <u>tell</u> what distribution Y has (and what is/are the parameter(s)).

VII. (28 points.) X and Y have joint probability mass function given in the table below.

$y \setminus x$	1	2	otherwise
0	$\frac{1}{10}$	$\frac{2}{10}$	0
1	$\frac{3}{10}$	$\frac{\frac{10}{4}}{10}$	0
otherwise			0
(1) (5 points) $E(XY) = ?$	I	I	

(2) (5 points) Cov(X, Y) = ?

- (3) (5 points) Are X and Y independent of each other? (give your arguments. no argument, no point.)
- (4) (8 points) Give $P\{Y = y \mid X = x\}$ for all values of x and y. (You need to organize them as a "table".)

(5) (5 points) $E\{Y \mid X = 1\} = ?$

VIII. (33 points.) X and Y have joint probability *density* function (PDF)

$$f(x,y) = \begin{cases} \frac{3}{4}y & 0 < x < 2, \ 0 < y < 2, \ x+y < 2\\ 0 & \text{otherwise.} \end{cases}$$

(1) (5 points) Give the (marginal) probability density (marginal PDF) $f_X(x)$ of X. (Be sure to specify the regions of x.)

(2) (5 points) Give the (marginal) probability density (marginal PDF) $f_Y(y)$ of Y. (Be sure to specify the regions of y.)

(3) (4 points) Are X and Y independent of each other? (give your arguments. no argument, no points!)

(4) (5 points) P(X > Y) =? (Leave your answer as an integral. Make sure your limits of integration are correct.)

(5) (6 points) Give the conditional probability density function (conditional PDF) of Y given that X = x. I.e., $f_{Y|X}(y \mid x) = ?$ (Be sure to specify the region of (x, y) in which your result holds.)

(6) (4 points) $E\left\{Y \mid X = \frac{1}{2}\right\} = ?$

(7) (4 points) $E\left\{(Y+1)X \mid X = \frac{1}{2}\right\} = ?$

Standard Normal Probabilities



Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998