

Math 447 Spring 2015

Exam 3

April 21, 2015

- Total value 220 points. Each part valued as indicated.
- SHOW YOUR WORK unless otherwise indicated. “NO WORK” may result in “NO POINTS”.
- Simplify your answers when possible. Do the arithmetic, remove parentheses, reduce fraction, etc.
- Cross out anything you don’t want graded!
- Use the back sides of pages if you need extra space. If you have anything on a back side that you want graded, indicate where it is.

Student: _____

Section (please circle): 01 (Xu)

Problem #	Possible Points	Points
I	30	
II	14	
III	21	
IV	36	
V	30	
VI	28	
VII	28	
VIII	33	
Total	220	

- I. (30 points.) X has probability mass function (PMF; i.e., $p(x) = P\{X = x\}$) given in the table below

x	-2	0	2	otherwise
$p(x) = P\{X = x\}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$	0

- (1) (8 points) Give the distribution function (CDF) of X .
I.e., $F(x) = P\{X \leq x\} = ?$

- (2) (6 points) $EX^4 = ?$

- (3) (6 points) $\text{Var } X = ?$

- (4) (6 points) Define $Y = \frac{1}{X+1}$. Give the moment generating function (MGF) of Y ,
 $m_Y(t) = ?$.

- (5) (4 points) Find the q th quantile of X with $q = \frac{1}{4}$. (No work need be shown.)

II. (14 points.) $\{X_1, \dots, X_n\}$ is an independent collection of random variables having finite mean μ and finite variance σ^2 . Let $S = \sum_{k=1}^3 kX_k$. Your answers to the questions below will involve μ and σ .

(1) (4 points.) $E(S) = ?$

(2) (4 points) $Var(S) = ?$

(3) (6 points) If $Y = X_1 + 2X_2 - 3$ and $Z = X_1 - 2X_2 + X_3 + 4$, then $Cov(Y, Z) = ?$

III. (21 points.) X has probability density function (PDF)

$$f_X(x) = \begin{cases} \frac{3x^2}{8} & 0 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$$

- (1) (6 points) Give the distribution function (CDF) of X .
I.e., $F(x) = P\{X \leq x\} = ?$ (Be sure to give all pieces of this function. Otherwise you will lose points.)

- (2) (6 points) $EX = ?$

- (3) (3 points) $P\{X = 1/3\} = ?$

- (4) (6 points) $P\{X^2 < \frac{1}{4}\} = ?$

IV. (36 points.) X has distribution function (CDF)

$$F_X(x) = \begin{cases} 0 & x < 0 \\ \frac{1+x^2}{10} & 0 \leq x < 2 \\ 1 & 2 \leq x \end{cases}$$

(1) (5 points) Does this distribution function (CDF) have any jumps? If yes, identify all the jump points and the associated probabilities (i.e. $P(X = x) = ?$).

(2) (5 points) $P\{-\pi < X \leq \frac{1}{2}\} = ?$

(3) (5 points) $P\{0 \leq x \leq 2\} = ?$

(4) (3 points) A is the set $\{-1, 0, \frac{1}{\sqrt{2}}, 2\}$. $P\{X \notin A\} = ?$

(5) (2 points) Find $F'(x) = ?$

(6) (8 points) $EX = ?$

(7) (8 points) $Var(X) = ?$ (give a summation of reduced fractions as your final answer, no need to simplify.)

V. (30 points.) X is normally distributed with mean $\mu = 3$ and variance $\sigma^2 = 36$.

(1) (5 points) $P\{3 \leq X \leq 15\} = ?$

(2) (5 points) $P\{|X - 6| \leq 9\} = ?$

(3) (5 points) Find out a value for x_0 so that $P\{X \geq x_0\} = 0.99$

(4) (5 points) $P\{X^3 + 27 \geq 0\} = ?$

(5) (5 points) Give the moment generating function (MGF) for X , $m_X(t) = ?$

(6) (5 points) Let $Y = 5X + 10$. Give the probability *density* function (PDF) for Y .
I.e., $f_Y(x) = ?$

VI. (28 points.) X has a *cumulative distribution function* (CDF). (If you don't pay attention to the difference between CDF and PDF, you can say goodbye to the following 33 pts)

$$F_X(x) = \begin{cases} 1 - e^{-5x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

(1) (4 points) $E(5X + 1) = ?$

(2) (4 points) $\text{Var}(5X + 1) = ?$

(3) (5 points) Give the moment generating function (MGF) of X , $m_X(t) = ?$

(4) (5 points) $E[(5X)^{20}] = ?$

(5) (5 points) $P\{X \leq \frac{1}{10} \mid X \geq \frac{1}{20}\} = ?$

(6) (5 points) Let $Y = 10X$. Give the distribution function (CDF) of Y . I.e., $F_Y(y) = ?$
Moreover, based on $F_Y(y)$, tell what distribution Y has (and what is/are the parameter(s)).

VII. (28 points.) X and Y have joint probability mass function given in the table below.

$y \backslash x$	1	2	otherwise
0	$\frac{1}{10}$	$\frac{2}{10}$	0
1	$\frac{3}{10}$	$\frac{4}{10}$	0
otherwise	0	0	0

(1) (5 points) $E(XY) = ?$

(2) (5 points) $Cov(X, Y) = ?$

(3) (5 points) Are X and Y independent of each other? (give your arguments. no argument, no point.)

(4) (8 points) Give $P\{Y = y \mid X = x\}$ for all values of x and y . (You need to organize them as a “table”.)

(5) (5 points) $E\{Y \mid X = 1\} = ?$

VIII. (33 points.) X and Y have joint probability *density* function (PDF)

$$f(x, y) = \begin{cases} \frac{3}{4}y & 0 < x < 2, 0 < y < 2, x + y < 2 \\ 0 & \text{otherwise.} \end{cases}$$

(1) (5 points) Give the (marginal) probability density (marginal PDF) $f_X(x)$ of X . (*Be sure to specify the regions of x .*)

(2) (5 points) Give the (marginal) probability density (marginal PDF) $f_Y(y)$ of Y . (*Be sure to specify the regions of y .*)

(3) (4 points) Are X and Y independent of each other? (give your arguments. no argument, no points!)

(4) (5 points) $P(X > Y) = ?$ (Leave your answer as an integral. Make sure your limits of integration are correct.)

(5) (6 points) Give the conditional probability density function (conditional PDF) of Y given that $X = x$. I.e., $f_{Y|X}(y | x) = ?$ (*Be sure to specify the region of (x, y) in which your result holds.*)

(6) (4 points) $E \{Y | X = \frac{1}{2}\} = ?$

(7) (4 points) $E \{(Y + 1)X | X = \frac{1}{2}\} = ?$

