Write in REF then RREF: $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 6 & 7 & 8 & 7 \end{bmatrix}$

$r_2 = R_2 - 5R_1$
$r_3 = R_3 - 6R_1$
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 0 & -5 & -10 & -17 \end{bmatrix}$$

scale
$r_2 = -\frac{1}{4}R_2$
$r_3 = -R_3$
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 5 & 10 & +17 \end{bmatrix}$$

$r_3 = R_3 - 5R_2$
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 2 \end{bmatrix} \longleftarrow \text{REF}$$

(<u>note</u>: REF is <u>not</u> unique, answers may vary)

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scale
$r_2 = \frac{1}{2}R_2$
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$r_1 = R_1 - 4R_3$
$r_2 = R_2 - 3R_3$
$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$r_1 = R_1 - 2R_2$
$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \longleftarrow \text{RREF}$$

(<u>note</u>: RREF is unique!)

Pivot columns: 1, 2, 4

Pivot positions: (1,1), (2,2), (3,4)

rank: 3

size: 3×4

$$\left[\begin{array}{cc|c} 1 & h & 3 \\ 2 & 8 & 1 \end{array}\right]$$ what value of h makes the system consistent/inconsistent?

$r_2 = R_2 - 2R_1$ $$\left[\begin{array}{cc|c} 1 & h & 3 \\ 0 & 8-2h & -5 \end{array}\right]$$

Since the position $(2,3)$ is nonzero in the augmented part of the matrix, the position $(2,2)$ must be nonzero for the system to have a solution.

$8-2h=0 \Rightarrow \underline{h=4}$ only value with no solutions

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$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 5 \\ 2 & 0 & 3 & 6 \end{array}\right]$$ Find all solutions of the SLE

$r_2 = R_2 - 2R_1$ $$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 5 \\ 0 & 0 & -1 & -4 \end{array}\right]$$ $r_2 = -R_2$ then $r_1 = R_1 - 2R_2$ $$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

Solution set $\{(-3,4)\}$

$$\left[\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{array}\right]$$

For which values of (g,h,k) is the system consistent.

$r_3 = R_3 + 2R_1$
$$\left[\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & -3 & 5 & k+2g \end{array}\right]$$

$r_3 = R_3 + R_2$
$$\left[\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & 0 & 0 & k+2g+h \end{array}\right]$$

this system reads

$$x - 4y + 7z = g$$
$$3y - 5z = h$$
$$\boxed{0 = k + 2g + h}$$

for consistency, this equation must be satisfied.

Parametric	Set
k, g anything $h = -k - 2g$	$\{(g,h,k) \mid k + 2g + h = 0\}$

$$\left[\begin{array}{cccc|c} 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \\ -2 & -5 & 8 & 0 & -17 \end{array}\right]$$

ⓐ which variables are basic/free?

ⓑ what is the rank of the augmented matrix?

$r_2 = R_2 - 3R_1$
$r_3 = R_3 - R_1$
$r_4 = R_4 + 2R_1$

$$\left[\begin{array}{cccc|c} 1 & 3 & -5 & 1 & 5 \\ 0 & 2 & -4 & 4 & -14 \\ 0 & 4 & -8 & 4 & -8 \\ 0 & 1 & -2 & 2 & -7 \end{array}\right]$$

$r_2 = R_4$
$r_4 = R_2$

$$\left[\begin{array}{cccc|c} 1 & 3 & -5 & 1 & 5 \\ 0 & 1 & -2 & 2 & -7 \\ 0 & 4 & -8 & 4 & -8 \\ 0 & 2 & -4 & 4 & -14 \end{array}\right]$$

$r_3 = R_3 - 4R_2$
$r_4 = R_4 - 2R_2$

$$\left[\begin{array}{cccc|c} 1 & 3 & -5 & 1 & 5 \\ 0 & 1 & -2 & 2 & -7 \\ 0 & 0 & 0 & -4 & 20 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

rank: 3

column 3 will be free

pivot column: 1, 2, 4 ← correspond to basic variables

$r_3 = -\frac{1}{4}R_3$

$$\left[\begin{array}{cccc|c} 1 & 3 & -5 & 1 & 5 \\ 0 & 1 & -2 & 2 & -7 \\ 0 & 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$r_2 = R_2 - 2R_3$
$r_1 = R_1 - R_3$

$$\left[\begin{array}{cccc|c} 1 & 3 & -5 & 0 & 10 \\ 0 & 1 & -2 & 0 & +3 \\ 0 & 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$r_1 = R_1 - 3R_2$

$$\begin{array}{c} \begin{array}{cccc} x & y & z & t \end{array} \\ \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -2 & 0 & -3 \\ 0 & 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

~~solution set~~ Parametric solution:

z anything

$t = -5$

$x = 1 - z$

$y = 2z - 3$