Quiz 9 MATH 304 Section 3 Name: Solution

To receive credit, the solution to problems will be clearly presented. Partial credit will not be awarded for problems worked without comments. Unwanted work should be completely erased or clearly scratched out.

1. Compute the determinant for each matrix either directly or through row reduction.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & -2 & 5 \\ 2 & 1 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 2 & 2 & 2 & 3 \\ -2 & -1 & -2 & -2 \\ 3 & 2 & 1 & 2 \end{bmatrix}$$

$$\det(A) = 1(-1)^{1+1}\det\left(\begin{bmatrix} -2 & 5 \\ 1 & 5 \end{bmatrix}\right) + 2(-1)^{1+3}\det\left(\begin{bmatrix} 3 & -2 \\ 2 & 1 \end{bmatrix}\right)$$ — 1st row expansion

$$= (-10-5) + 2(3+4) = -15+14 = \underline{\underline{-1}}$$

$$\begin{bmatrix} 2 & 1 & 1 & 1 \\ 2 & 2 & 2 & 3 \\ -2 & -1 & -2 & -2 \\ 3 & 2 & 1 & 2 \end{bmatrix} \xrightarrow[\text{then } r_1 \leftrightarrow r_4]{r_4 = R_4 - R_1} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 2 & 2 & 2 & 3 \\ -2 & -1 & -2 & -2 \\ 2 & 1 & 1 & 1 \end{bmatrix} \xrightarrow[\substack{r_3 = R_3 + 2R_1 \\ r_4 = R_4 - 2R_1}]{r_2 = R_2 - 2R_1} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 1 & -2 & 0 \\ 0 & -1 & 1 & -1 \end{bmatrix} \xrightarrow[\text{then } r_2 \leftrightarrow r_3]{r_4 = R_4 + R_3}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & -1 & -1 \end{bmatrix} \xrightarrow[\text{then } r_3 \leftrightarrow r_4]{r_3 = R_3 + 2R_4} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & -1 \end{bmatrix} \leftarrow \overline{B}$$

$$E_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} \quad E_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad E_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad E_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -2 & 0 & 0 & 1 \end{bmatrix} \quad E_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad E_7 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad E_8 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_9 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\det(B) = \frac{\det(\overline{B})}{\det(E_1)\cdots\det(E_9)}$$

$$= \frac{1(1)(-1)(-1)}{1(-1)(1)(1)(1)(1)(-1)(1)(-1)}$$

$$= \frac{1}{-1} = -1$$

2. For what values of t is the matrix $\begin{bmatrix} t & 2 \\ 1 & t-1 \end{bmatrix}$ singular?

$$\det\left(\begin{bmatrix} t & 2 \\ 1 & t-1 \end{bmatrix}\right) = t(t-1) - 2 = t^2 - t - 2 = (t-2)(t+1)$$

$\begin{bmatrix} t & 2 \\ 1 & t-1 \end{bmatrix}$ is singular only when $(t-2)(t+1) = 0$

or when $t \in \{-1, 2\}$.