Quiz 8 MATH 304 Section 3 Name: Solution

To receive credit, the solution to problems will be clearly presented. Partial credit will not be awarded for problems worked without comments. Unwanted work should be completely erased or clearly scratched out.

1. Suppose that $\mathcal{X} = (1+x, 2+x)$ and $\mathcal{Y} = (1-x, 2-x)$ are bases for the vector space $\mathbb{P}_1$.

   (a) Calculate the change of basis matrices ${}_{\mathcal{X}}I_{\mathcal{Y}}$ and ${}_{\mathcal{Y}}I_{\mathcal{X}}$.

   (b) Suppose that $\mathcal{S} = (1, x)$ is a basis for $\mathbb{P}_1$ and that ${}_{\mathcal{X}}F_{\mathcal{Y}} = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$. Find the matrix ${}_{\mathcal{S}}F_{\mathcal{S}}$.

(a)

${}_{\mathcal{X}}I_{\mathcal{Y}} = [K_{\mathcal{X}}(1-x) \quad K_{\mathcal{X}}(2-x)]$ the solution to $\left[\begin{array}{cc|cc} 1 & 2 & 1 & 2 \\ 1 & 1 & -1 & -1 \end{array}\right]$

$= \begin{bmatrix} -3 & -4 \\ 2 & 3 \end{bmatrix}$

${}_{\mathcal{Y}}I_{\mathcal{X}} = [K_{\mathcal{Y}}(1+x) \quad K_{\mathcal{Y}}(2+x)]$ the solution to $\left[\begin{array}{cc|cc} 1 & 2 & 1 & 2 \\ -1 & -1 & 1 & 1 \end{array}\right]$

$= \begin{bmatrix} -3 & -4 \\ 2 & 3 \end{bmatrix}$

(also note that ${}_{\mathcal{X}}I_{\mathcal{Y}} = ({}_{\mathcal{Y}}I_{\mathcal{X}})^{-1}$)

(b) ${}_{\mathcal{S}}F_{\mathcal{S}} = ({}_{\mathcal{S}}I_{\mathcal{X}})({}_{\mathcal{X}}F_{\mathcal{Y}})({}_{\mathcal{Y}}I_{\mathcal{S}})$

${}_{\mathcal{S}}I_{\mathcal{X}} = [K_{\mathcal{S}}(1+x) \quad K_{\mathcal{S}}(2+x)]$

$= \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$

${}_{\mathcal{Y}}I_{\mathcal{S}} = [K_{\mathcal{Y}}(1) \quad K_{\mathcal{Y}}(x)]$ the solution to $\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ -1 & -1 & 0 & 1 \end{array}\right]$

$= \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow {}_{\mathcal{S}}F_{\mathcal{S}} = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -3 & -9 \\ -2 & -6 \end{bmatrix}$

$F(2x-1) = K_{\mathcal{X}}^{-1}({}_{\mathcal{X}}F_{\mathcal{Y}})K_{\mathcal{Y}}(2x-1) = K_{\mathcal{S}}^{-1}({}_{\mathcal{S}}F_{\mathcal{S}})K_{\mathcal{S}}(2x-1)$

$= K_{\mathcal{S}}^{-1}\left(\begin{bmatrix} -3 & -9 \\ -2 & -6 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix}\right)$

$= K_{\mathcal{S}}^{-1}\left(\begin{bmatrix} -15 \\ -10 \end{bmatrix}\right)$

2. **Bonus (2 points)** Use either ${}_{\mathcal{X}}F_{\mathcal{Y}}$ or ${}_{\mathcal{S}}F_{\mathcal{S}}$ to calculate $F(2x-1)$.

$= -15 - 10x$