Quiz 7 MATH 304 Section 3 Name: Solution

To receive credit, the solution to problems will be clearly presented. Partial credit will not be awarded for problems worked without comments. Unwanted work should be completely erased or clearly scratched out.

1.

$$\mathcal{X} = \{\ 3x^2 - 1\ ,\ -3x^2 - 2x\ ,\ 6x^2 + 2x + 1\ ,\ 6x^2 - 2\ \}$$

(i) Find a basis $\mathcal{B}$ for the subspace of $\mathbb{P}_2$ spanned by $\mathcal{X}$.

(ii) Find a basis $\mathcal{C}$ for V which contains $\mathcal{B}$.

using $\mathcal{Y} = (x^2, x, 1)$

$$K_{\mathcal{Y}}(\mathcal{X}) = \left\{ \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -3 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ -2 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 3 & -3 & 6 & 6 \\ 0 & -2 & 2 & 0 \\ -1 & 0 & 1 & -2 \end{bmatrix} \begin{matrix} r_1 = \frac{1}{3}R_1 \\ r_2 = -\frac{1}{2}R_2 \end{matrix} \begin{bmatrix} 1 & -1 & 2 & 2 \\ 0 & +1 & -1 & 0 \\ -1 & 0 & 1 & -2 \end{bmatrix} r_3 = R_3 + R_1 \begin{bmatrix} 1 & -1 & 2 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 3 & 0 \end{bmatrix}$$

$$r_3 = R_3 + R_2 \begin{bmatrix} 1 & -1 & 2 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

(i) let $\mathcal{B} = \{3x^2-1, -3x^2-2x, 6x^2+2x+1\}$

$\mathcal{B}$ is a basis for Span($\mathcal{X}$) as

(i) span($\mathcal{B}$) = span($\mathcal{X}$) (span preservation)

(ii) $\mathcal{B}$ is LI as the rank($\mathcal{B}$) = # elements

(ii) $\dim(\mathbb{P}_2) = 3$, so the span($\mathcal{B}$) = $\mathbb{P}_2$

let $\mathcal{C} = \mathcal{B}$.

2. **Clearly** circle "True" or "False" for each of the following problems. Circle "True" only if the statement is always true. No explanation is necessary.

TRUE **[FALSE]** (a) A single vector is always linearly independent. $\{\vec{0}\}$ is LD

[TRUE] FALSE (b) If a finite set S of nonzero vectors spans a vector space V, then some subset of S is a basis for V.

[TRUE] FALSE (c) A basis is a linearly independent set that is as large as possible.

[TRUE] FALSE (d) The dimension of the subspace spanned by a set of vectors $\mathcal{X}$ is the rank of the matrix for $\mathcal{X}$.

[TRUE] FALSE (e) The only three-dimensional subspace of $\mathbb{R}^3$ is $\mathbb{R}^3$ itself.

TRUE **[FALSE]** (f) The vector space of polynomials with degree at most four, $\mathbb{P}_4$, has dimension 4.

$\dim(\mathbb{P}_4) = 5$