Quiz 3 MATH 304 Section 5 Name: *Solution*

To receive credit, the solution to problems will be clearly presented. Partial credit will not be awarded for problems worked without comments. Unwanted work should be completely erased or clearly scratched out.

1.
$$A = \begin{bmatrix} 1 & 2 & 4 & 1 \\ 2 & 1 & 1 & 1 \\ -1 & 4 & 10 & 1 \end{bmatrix} \quad \begin{matrix} r_2 = R_2 - 2R_1 \\ r_3 = R_3 + R_1 \end{matrix} \quad \begin{bmatrix} 1 & 2 & 4 & 1 \\ 0 & -3 & -7 & 1 \\ 0 & 6 & 14 & 2 \end{bmatrix}$$

(a) Find a basis for col($A$). What is dim(col($A$))?

(b) Find a basis for nul($A$). What is dim(nul($A$))?

(c) Find a basis for row($A$). What is dim(row($A$))?

$$r_3 = R_3 + 2R_2 \quad \begin{bmatrix} 1 & 2 & 4 & 1 \\ 0 & -3 & -7 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$r_3 = -\tfrac{1}{3}R_3 \text{ then } r_1 = R_1 - 2R_2 \quad \begin{bmatrix} 1 & 0 & -2/3 & 1/3 \\ 0 & 1 & 7/3 & 1/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) A basis for Col(A) are the pivots

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} \right\} \quad \dim(\text{col}(A)) = 2$$

(b) A basis for row(A) is $\{[1\ 2\ 4\ 1], [2\ 1\ 1\ 1]\}$ $\dim(\text{row}(A)) = 2$

(c) ~~A~~ solution to $A\vec{x} = \vec{0}$ is of the form

dim(nul(A))

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 2/3c - 1/3d \\ -7/3c - 1/3d \\ c \\ d \end{bmatrix} = c\begin{bmatrix} 2/3 \\ -7/3 \\ 1 \\ 0 \end{bmatrix} + d\begin{bmatrix} -1/3 \\ -1/3 \\ 0 \\ 1 \end{bmatrix}$$

A basis for nul (A) is $\left\{ \begin{bmatrix} 2/3 \\ -7/3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1/3 \\ -1/3 \\ 0 \\ 1 \end{bmatrix} \right\}$ or we can take $\left\{ \begin{bmatrix} 2 \\ -7 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 0 \\ 3 \end{bmatrix} \right\}$

2. Assume that $A$ is a $5 \times 9$ matrix. Justify your answers.

(a) If $A$ has two pivot columns, what is dim(col($A$)) and dim(nul($A$))?

(b) Is it possible for $A$ to be onto? If no, explain why. If yes, what is dim(col($A$)) and dim(nul($A$))?

(c) Is it possible for $A$ to be 1-to-1? If no, explain why. If yes, what is dim(col($A$)) and dim(nul($A$))?

(a) $\dim(\text{col}(A)) = 2$ and $\dim(\text{nul}(A)) = 9 - 2 = 7$

(b) onto $\Rightarrow \dim(\text{col}(A)) = 5$ so $\dim(\text{nul}(A)) = 9 - 5 = 4$ YES

(c) 1-to-1 $\Rightarrow \dim(\text{nul}(A)) = 0 \Rightarrow \dim(\text{col}(A)) = 9$ NO the dim of the codomain is only 5!

3. **Bonus:** Verify that the basis vectors found in 1($a$) and 1($b$) are linearly independent.