Quiz 4 MATH 304 Section 3 Name: Solution

To receive credit, the solution to problems will be clearly presented. Partial credit will not be awarded for problem [w]orked without comments. Unwanted work should be completely erased or clearly scratched out.

1. (i) For a matrix A, define the Null space of A.

$Nul(A) = \{\vec{w} \in \mathbb{R}^n \mid A\vec{w} = \vec{0}\}$

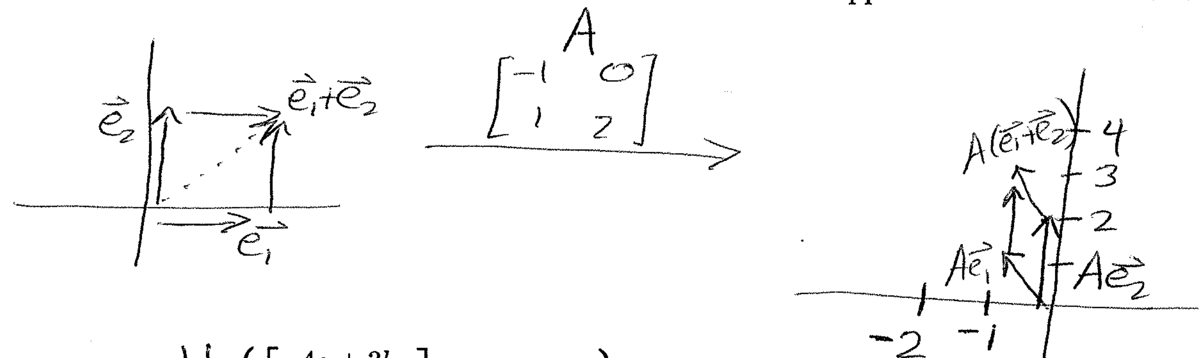
in other words, $Nul(A)$ consists of vectors which multiplied to A result in $\vec{0}$.

(ii) Is $\vec{w} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$ in the Null space of $\begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix}$?

NO

$$\begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} = 1\begin{bmatrix} 3 \\ 6 \\ -8 \end{bmatrix} + 3\begin{bmatrix} -5 \\ -2 \\ 4 \end{bmatrix} + 4\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ -8 \end{bmatrix} + \begin{bmatrix} -15 \\ -6 \\ 12 \end{bmatrix} + \begin{bmatrix} -12 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} -24 \\ 0 \\ 8 \end{bmatrix}$$

2. Use standard basis vectors to visualize how the domain is mapped to the codomain for the matrix $\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$.



3. Is the subset $H = \left\{ \begin{bmatrix} 4a+3b \\ 0 \\ a+b+c \\ c-2a \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$ a subspace of $\mathbb{R}^4$? Justify your answer.

YES

(a)

$$\underset{\in H}{\begin{bmatrix} 4a+3b \\ 0 \\ a+b+c \\ c-2a \end{bmatrix}} + \underset{\in H}{\begin{bmatrix} 4d+3e \\ 0 \\ d+e+f \\ f-2d \end{bmatrix}} = \underset{\in H}{\begin{bmatrix} 4(a+d)+3(b+e) \\ 0 \\ (a+d)+(b+e)+(c+f) \\ (c+f)-2(a+d) \end{bmatrix}}$$

(b)

$$d\begin{bmatrix} 4a+3b \\ 0 \\ a+b+c \\ c-2a \end{bmatrix} = \begin{bmatrix} 4(ad)+3(bd) \\ 0 \\ (ad)+(bd)+(cd) \\ (cd)-2(ad) \end{bmatrix} \in H$$

4. Let $\vec{v_1} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\vec{v_2} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, $\vec{v_3} = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} 8 \\ 4 \\ 7 \end{bmatrix}$.

(i) Is $\vec{w}$ in Span$\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$?

(ii) If yes, then what are the coefficients for $\vec{w}$ in the linear combination of $\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$?

(i)

Solve

$\vec{w} \notin Span(\{\vec{v_1}, \vec{v_2}, \vec{v_3}\})$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 8 \\ 0 & 1 & 2 & 4 \\ -1 & 3 & 6 & 7 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 2 & 4 & 8 \\ 0 & 1 & 2 & 4 \\ 0 & 5 & 10 & 15 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 2 & 4 & 8 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & -5 \end{array}\right]$$

$r_3 = R_3 + R_1$ $\qquad$ $r_3 = R_3 - 5R_2$ $\qquad$ Inconsistent!