Quiz 2 MATH 304 Section 3 Name: Solution

To receive credit, the solution to problems will be clearly presented. Partial credit will not be awarded for problem worked without comments. Unwanted work should be completely erased or clearly scratched out.

1. Let the function $H : \mathbb{R}^3 \to \mathbb{R}^3$ be defined

$$H\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ x_1 + x_2 + x_3 \\ x_1 \end{bmatrix}$$

Note: This function cannot be represented as a matrix.

a) Is the function H a linear transformation? Justify your answer.

b) The function is neither onto nor 1-to-1. Justify this claim.

(a) 1) $\mathbb{R}^3$ is a vector space.

2) $$H\left(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}\right) = H\left(\begin{bmatrix} a_1+b_1 \\ a_2+b_2 \\ a_3+b_3 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ a_1+b_1+a_2+b_2+a_3+b_3 \\ a_1+b_1 \end{bmatrix}$$

$$H\left(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}\right) + H\left(\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ a_1+a_2+a_3 \\ a_1 \end{bmatrix} + \begin{bmatrix} 1 \\ b_1+b_2+b_3 \\ b_1 \end{bmatrix} =$$

$$\begin{bmatrix} 2 \\ a_1+b_1+a_2+b_2+a_3+b_3 \\ a_1+b_1 \end{bmatrix}$$

$$H\left(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}\right) \neq H\left(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}\right) + H\left(\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}\right)$$ NOT linear transformation

(b) Not onto as $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ isn't an output of H.

Not 1-to-1 as $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ both are taken to $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ by H.

2. Definitions: Define the following bold terms from Chapter 1. A definition is a general statement and NOT an example.

a) The **rank** of a matrix M. The # of pivot columns of M when in REF.

b) A **1-to-1** function $F : V \to W$. A function where $F(v) = F(w)$ implies $v = w$.

c) An **onto** function $F : V \to W$. A function where for every $w \in W$ there is at least one $v \in V$ where $F(v) = w$.

d) A **homogeneous** system of linear equations. A system whose augmented matrix has a zero column in the right column.