

To receive credit, the solution to problems will be clearly presented. Partial credit will not be awarded for problems worked without comments. Unwanted work should be completely erased or clearly scratched out.

1. Solve the system of linear equations

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + x_4 &= 8 \\ x_1 + 2x_3 + x_4 &= 3 \\ x_1 + 3x_2 + x_4 &= 7 \end{aligned}$$

If the system is consistent, write the solution as a set or in parametric form.

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 8 \\ 1 & 0 & 2 & 1 & 3 \\ 1 & 3 & 0 & 1 & 7 \end{array} \right] \begin{array}{l} R_2 = R_2 - R_1 \\ R_3 = R_3 - R_1 \end{array} \quad \left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 8 \\ 0 & -2 & -1 & 0 & -5 \\ 0 & 1 & -3 & 0 & -1 \end{array} \right] \begin{array}{l} R_2 = R_3 \\ R_3 = R_2 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & -2 & -1 & 0 & -5 \end{array} \right] R_3 = R_3 + 2R_2 \quad \left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & -7 & 0 & -7 \end{array} \right] R_3 = -\frac{1}{7}R_3$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \end{array} \right] \begin{array}{l} R_2 = R_2 + 3R_3 \\ R_1 = R_1 - 3R_3 \end{array} \quad \left[\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 5 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \end{array} \right] R_1 = R_1 - 2R_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \end{array} \right] \begin{array}{l} L_{\text{free}} \\ x_1 = 1 - x_4 \\ x_2 = 2 \\ x_3 = 1 \end{array} \quad \boxed{\begin{array}{l} x_1 = 1 - x_4 \\ x_2 = 2 \\ x_3 = 1 \end{array} \quad x_4 \text{ is anything}}$$

2. Which of the matrices below are in row echelon form? If not, why not? Which are in reduced row echelon form? If not, why not?

REF
B, C

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 2 & 3 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{pmatrix}$$

Why not?

A → zero row not below non-zero rows

$$C = \begin{pmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$D = \begin{pmatrix} 1 & 3 & 0 & 2 & 0 \\ 0 & 0 & 2 & 2 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

B → not in RREF

due to scaling in pivot ~~column~~ position and multiple nonzeros in pivot columns.

RREF
C

D → not in REF as leading entry row 2 is left of that in row 3.