Homework 8 MATH 304 Section 3 Solution

Assigned: Wednesday, October 8.
Potentially Collected: Wednesday, October 15.

1. Find the standard basis matrix for the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that maps a point (x_1, x_2) into

   (a) its reflection about the line $x_2 = -x_1$.

   (b) its projection on the x_1-axis.

   (c) its projection on the x_2-axis.

   (d) its rotation clockwise through and angle of θ.

(a) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ reflect about the line $y = -x$ (aka $x_2 = -x_1$)

$A\vec{e}_1 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ $A\vec{e}_2 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$

2. For each of the matrices below, use standard basis vectors to visualize how the domain is mapped to the codomain.

   (a) $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$

   (b) $\begin{bmatrix} 3 \\ -2 \\ -6 \end{bmatrix}$

   (c) $\begin{bmatrix} -1 & 2 \end{bmatrix}$

   (d) $\begin{bmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}$

   (e) $\begin{bmatrix} -1 & 2 \\ 2 & -4 \end{bmatrix}$

$A\vec{e}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $A\vec{e}_2 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$B\vec{e}_1 = \begin{bmatrix} 3 \\ -2 \\ -6 \end{bmatrix}$

$C\vec{e}_1 = [-1]$ $C\vec{e}_2 = [2]$

(b) Projection sends (x_1, x_2) to $(x_1, 0)$

$B\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $B\vec{e}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$D\vec{e}_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ $D\vec{e}_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

$E\vec{e}_1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ $E\vec{e}_2 = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$

(c) Projection sends (x_1, x_2) to $(0, x_2)$

$C\vec{e}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $C\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

(d) $\theta \leftarrow$ counterclockwise rotation (see class notes)

$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

a clockwise rotation is $-\theta$ of counterclockwise $\begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix}$