

Homework 7

Solutions:

1) (a) If column 1 of A is a zero column, then the 1st row of B has a zero coefficient when creating linear combinations of B 's rows.

(b) If row 1 of B is a zero row, then the 1st column of A has a zero coefficient when creating linear combinations of A 's columns.

$$2) \vec{0} = AB^T = [1 \ r \ 1] \begin{bmatrix} -2 \\ 2 \\ s \end{bmatrix} = [1(-2) + r(2) + 1(s)] = [2r + s - 2]$$

$$\Rightarrow 2r + s - 2 = 0$$

$$\Rightarrow s = 2 - 2r$$

r anything

3) a) REF for A $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ RREF for A $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

b) $\text{Rank}(A) = 3$

c) A is not onto

d) A is not 1-to-1

$$\begin{bmatrix} 2 & 2 & 2 & 2 & : & a \\ -4 & 0 & 0 & -4 & : & b \\ 3 & 3 & 2 & 2 & : & c \\ 2 & 4 & 2 & 0 & : & d \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & : & \frac{1}{2}a \\ 0 & 1 & 1 & 0 & : & \frac{1}{4}b + \frac{1}{2}a \\ 0 & 0 & 1 & 1 & : & \frac{3}{2}a - c \\ 0 & 0 & 0 & 0 & : & 3a - \frac{1}{2}b - 2c - d \end{bmatrix}$$

for (c) the vector

$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ is not in the image.

for (d) the vectors $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ are taken to $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ by A .