Homework 4 MATH 304 Section 3

Assigned: Friday, September 12.
Potentially Collected: Friday, September 19.

1. Let $\vec{u} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$. Compute the following vectors:

   a) $\vec{u}+\vec{v}$.

   b) $\vec{u}-\vec{v}$.

   c) $\vec{0}-3\vec{v}$.

   d) $2\vec{u}-3\vec{v}$.

(a) $\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 7 \end{bmatrix}$

(c) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - 3\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} -3 \\ -6 \\ 9 \end{bmatrix}$

(d) $2\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} - 3\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 8 \end{bmatrix} + \begin{bmatrix} -3 \\ -6 \\ 9 \end{bmatrix} = \begin{bmatrix} 1 \\ -8 \\ 17 \end{bmatrix}$

2. Which of the following functions are linear transformations?

   a) $F : \mathbb{R}^2 \to \mathbb{R}^3$ where

   $$F\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1+1 \\ x_2 \\ x_1+x_2 \end{bmatrix}$$

   b) $G : \mathbb{R}^2 \to \mathbb{R}^3$ where

   $$G\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1+x_2 \\ x_2 \\ x_1-x_2 \end{bmatrix}$$

   c) $H : \mathbb{R}^3 \to \mathbb{R}^3$ where

   $$H\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_2^2+x_3^2 \\ x_3^2 \end{bmatrix}$$

Verify: 1) $F(\vec{x}+\vec{y}) = F(\vec{x}) + F(\vec{y})$
$cF(\vec{x}) = F(c\vec{x})$

(a) $F\left(\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}\right) = F\left(\begin{bmatrix} a+c \\ b+d \end{bmatrix}\right) = \begin{bmatrix} a+c+1 \\ b+d \\ a+b+c+d \end{bmatrix}$

$F\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) + F\left(\begin{bmatrix} c \\ d \end{bmatrix}\right) = \begin{bmatrix} a+1 \\ b \\ a+b \end{bmatrix} + \begin{bmatrix} c+1 \\ d \\ c+d \end{bmatrix} = \begin{bmatrix} a+c+2 \\ b+d \\ a+b+c+d \end{bmatrix} \neq F\left(\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}\right)$

<u>NOT</u> Linear Transformation

<u>NOT</u> Linear Transformation

(b) $G\left(\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}\right) = G\left(\begin{bmatrix} a+c \\ b+d \end{bmatrix}\right) = \begin{bmatrix} a+b+c+d \\ b+d \\ a+c-b-d \end{bmatrix}$

$G\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) + G\left(\begin{bmatrix} c \\ d \end{bmatrix}\right) = \begin{bmatrix} a+b \\ b \\ a-b \end{bmatrix} + \begin{bmatrix} c+d \\ d \\ c-d \end{bmatrix} =$ ✓

$cG\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = c\begin{bmatrix} a+b \\ b \\ a-b \end{bmatrix} = \begin{bmatrix} ca+cb \\ cb \\ ca-cb \end{bmatrix} = G\left(c\begin{bmatrix} a \\ b \end{bmatrix}\right)$

✓ G <u>is</u> a Linear Transformation.

(c) $H\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}\right) = H\left(\begin{bmatrix} x_1+y_1 \\ x_2+y_2 \\ x_3+y_3 \end{bmatrix}\right) = \begin{bmatrix} x_1+y_1 \\ (x_2+y_2)^2+(x_3+y_3)^2 \\ (x_3+y_3)^2 \end{bmatrix} = \begin{bmatrix} x_1+y_1 \\ x_2^2+x_3^2+y_2^2+y_3^2+2(x_2y_2+x_3y_3) \\ x_3^2+y_3^2+2x_3y_3 \end{bmatrix}$

$= \begin{bmatrix} x_1 \\ x_2^2+x_3^2 \\ x_3^2 \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2^2+y_3^2 \\ y_3^2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2(x_2y_2+x_3y_3) \\ 2x_3y_3 \end{bmatrix} = H\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) + H\left(\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}\right) + \begin{bmatrix} 0 \\ 2x_2y_2+2x_3y_3 \\ 2x_3y_3 \end{bmatrix}$