Homework 3 MATH 304 Section 3

Assigned: Wednesday, September 10.
Potentially Collected: Wednesday, September 17.

1. Find an equation relating a, b, and c so that the linear system

$$2x + 2y + 3z = a$$
$$3x - y + 5z = b$$
$$x - 3y + 2z = c$$

is inconsistent for any values of a, b, and c that satisfy that equation.

2. Determine the reduced row echelon form of

$$\begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}$$

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(1) Augmented Matrix

$$\left[\begin{array}{ccc|c} 2 & 2 & 3 & a \\ 3 & -1 & 5 & b \\ 1 & -3 & 2 & c \end{array}\right] \begin{array}{l} r_1 = R_3 \\ r_3 = R_1 \end{array} \left[\begin{array}{ccc|c} 1 & -3 & 2 & c \\ 3 & -1 & 5 & b \\ 2 & 2 & 3 & a \end{array}\right] \begin{array}{l} r_2 = R_2 - 3R_1 \\ r_3 = R_3 - 2R_1 \end{array} \left[\begin{array}{ccc|c} 1 & -3 & 2 & c \\ 0 & 8 & -1 & b-3c \\ 0 & 8 & -1 & a-2c \end{array}\right]$$

$$r_3 = R_3 - R_2 \left[\begin{array}{ccc|c} 1 & -3 & 2 & c \\ 0 & 8 & -1 & b-3c \\ 0 & 0 & 0 & a-2c-b+3c \end{array}\right]$$

to be consistent only if $a - b + c = 0$
inconsistent otherwise.

(2)
$$\left[\begin{array}{cc} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{array}\right] r_1 = \frac{1}{\cos(\theta)} R_1 \left[\begin{array}{cc} 1 & \tan(\theta) \\ -\sin(\theta) & \cos(\theta) \end{array}\right] r_2 = R_2 + \sin(\theta) R_1$$

$$\left[\begin{array}{cc} 1 & \tan(\theta) \\ 0 & \cos(\theta) + \sin(\theta)\tan(\theta) \end{array}\right] = \left[\begin{array}{cc} 1 & \tan(\theta) \\ 0 & \sec(\theta) \end{array}\right] \begin{array}{l} r_2 = \cos(\theta) R_2 \\ \text{then} \\ r_1 = R_1 - \tan(\theta) R_2 \end{array} \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right]$$

$$\cos(\theta) + \frac{\sin(\theta)\sin(\theta)}{\cos(\theta)} = \frac{\cos^2(\theta) + \sin^2(\theta)}{\cos(\theta)} = \frac{1}{\cos(\theta)} = \sec(\theta)$$