Homework 2 MATH 304 Section 3

Assigned: Monday, September 8.
Potentially Collected: Monday, September 15.

1. Find the general solution of the system

$$x_1 - 2x_2 - x_3 + 3x_4 = 0$$
$$-2x_1 + 4x_2 + 5x_3 - 5x_4 = 3$$
$$3x_1 - 6x_2 - 6x_3 + 8x_4 = 2$$

2. Suppose that a 3×5 coefficient matrix for a system has three pivot columns. Is the system consistent? Why or why not?

3. Suppose that a system of linear equations has a 3×5 augmented matrix whose fifth column is a pivot column. Is the system consistent? Why or why not?

4. Suppose the coefficient matrix of a system of linear equations has a pivot position in every row. Explain why the system is consistent.

(1) System as Matrix, row reduce to RREF

$$\left[\begin{array}{cccc|c} 1 & -2 & -1 & 3 & 0 \\ -2 & 4 & 5 & -5 & 3 \\ 3 & -6 & -6 & 8 & 2 \end{array}\right] \quad r_2 = R_2 + 2R_1, \; r_3 = R_3 - 3R_1 \quad \left[\begin{array}{cccc|c} 1 & -2 & -1 & 3 & 0 \\ 0 & 0 & 3 & 1 & 3 \\ 0 & 0 & -3 & -1 & 2 \end{array}\right]$$

$$r_3 = R_3 + R_2 \quad \left[\begin{array}{cccc|c} 1 & -2 & -1 & 3 & 0 \\ 0 & 0 & 3 & 1 & 3 \\ 0 & 0 & 0 & 0 & 5 \end{array}\right] \quad 0 = 5?$$

Inconsistent!
No solution
(Augment is a pivot!)

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(2)

$$\begin{array}{c} \\ 3 \text{ rows} \end{array}\left[\begin{array}{ccccc|c} & & & & & \\ & & C & & & A \\ & & & & & \\ \multicolumn{5}{c}{\underbrace{\qquad\qquad\qquad}_{3 \text{ pivots}}} & \end{array}\right]$$

5 columns, 6th column

The rank of this matrix cannot exceed 3, the number of rows. As 3 pivots are in C, A cannot be a pivot! Consistent!!!

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(3) Inconsistent, as the augmented column is a pivot.

(4) The number of pivots is $\leq$ the number of rows, so the augmented column isn't a pivot.