

# Homework 23 MATH 304 Section 3

Solution

Assigned: Friday, November 21.  
Potentially Collected: Monday, December 1.

1. Find the characteristic polynomials for each of the following matrices.

(a)  $\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} = A$       $A - \lambda I_2 = \begin{bmatrix} 2-\lambda & 1 \\ -1 & 3-\lambda \end{bmatrix}$

(b)  $\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = B$       $\det(A - \lambda I_2) = (2-\lambda)(3-\lambda) - (-1)$

(c)  $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ -1 & 3 & 2 \end{bmatrix} = C$       $= 6 - 5\lambda + \lambda^2 + 1 = \lambda^2 - 5\lambda + 7$

(d)  $\begin{bmatrix} 4 & -1 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix} = D$       $B - \lambda I_2 = \begin{bmatrix} 4-\lambda & 2 \\ 3 & 3-\lambda \end{bmatrix}$

$\det(B - \lambda I_2) = (4-\lambda)(3-\lambda) - 6$   
 $= 12 - 7\lambda + \lambda^2 - 6 = \lambda^2 - 7\lambda + 6$

$C - \lambda I_3 = \begin{bmatrix} 1-\lambda & 2 & 1 \\ 0 & 1-\lambda & 2 \\ -1 & 3 & 2-\lambda \end{bmatrix}$

2<sup>nd</sup> row expansion

$\det(C - \lambda I_3) = (1-\lambda)(-1)^{2+2} \det\left(\begin{bmatrix} 1-\lambda & 1 \\ -1 & 2-\lambda \end{bmatrix}\right) + 2(-1)^{2+3} \det\left(\begin{bmatrix} 1-\lambda & 2 \\ -1 & 3 \end{bmatrix}\right)$

$= (1-\lambda)[(1-\lambda)(2-\lambda) + 1] - 2[(1-\lambda)3 + 2]$

$= (1-\lambda)[2 - 3\lambda + \lambda^2 + 1] - 2[3 - 3\lambda + 2] = (1-\lambda)(\lambda^2 - 3\lambda + 3) - 2(-3\lambda + 5)$

$= \lambda^2 - 3\lambda + 3 - \lambda^3 + 3\lambda^2 - 3\lambda + 6\lambda - 10 = -\lambda^3 + 4\lambda^2 - 7$

$D - \lambda I_3 = \begin{bmatrix} 4-\lambda & -1 & 3 \\ 0 & 2-\lambda & 1 \\ 0 & 0 & 3-\lambda \end{bmatrix}$

$\det(D - \lambda I_3) = (4-\lambda)(2-\lambda)(3-\lambda)$

determinant of REF matrices is the product of the main diagonal

$= (8 - 6\lambda + \lambda^2)(3 - \lambda) = 24 - 18\lambda + 3\lambda^2 - 8\lambda + 6\lambda^2 - \lambda^3$

$= -\lambda^3 + 9\lambda^2 - 26\lambda + 24$

Eigenvalues {4, 2, 3}