

# Homework 21 MATH 304 Section 3

Solution

Assigned: Monday, November 17.  
 Potentially Collected: Monday, November 24.

1. Find the determinant of the following matrices:

$$\det \begin{pmatrix} 0 & 5 & 1 \\ 4 & -3 & 0 \\ 2 & 4 & 1 \end{pmatrix} = 5(-1)^{1+2} \det \begin{pmatrix} 4 & 0 \\ 2 & 1 \end{pmatrix} + 1(-1)^{1+3} \det \begin{pmatrix} 4 & -3 \\ 2 & 4 \end{pmatrix}$$

$$= -5(4) + (16+6) = -20 + 22 = \underline{\underline{2}}$$

$$\det \begin{pmatrix} 4 & 0 & 0 & 0 \\ 7 & -1 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 5 & -8 & 4 & -3 \end{pmatrix} = 4(-1)3(-3) = 36$$

product of the diagonals as the matrix is triangular.

$$\det \begin{pmatrix} 6 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 2 & 0 & 0 & 0 \\ 8 & 3 & 1 & 8 \end{pmatrix} = 2(-1)^{3+1} \det \begin{pmatrix} 6 & 0 & 5 \\ 7 & 2 & -5 \\ 8 & 3 & 8 \end{pmatrix}$$

— 3<sup>rd</sup> row expansion  
— 1<sup>st</sup> row expansion

$$= 2(5(-1)^{1+3} \det \begin{pmatrix} 7 & 2 \\ 8 & 1 \end{pmatrix}) = 10(7-6) = 10$$

2. Compute the determinants of the elementary matrices:

$$\det \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ k & 0 & 1 \end{pmatrix} = 1^3 = 1$$

— product of the diagonals as the matrix is triangular

$$\det \begin{pmatrix} k & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = k$$

$$\det \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = (-1)^{1+2} \det \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = -1(1-0) = \underline{\underline{-1}}$$