Homework 13 MATH 304 Section 3

Solution

Assigned: Monday, October 20.
Potentially Collected: Monday, October 27.

1. For each of the following matrix, find a basis for the Null Space, Row Space, and the Column Space. What is the dimension of each of the subspaces?

$$M = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & -5 & 4 \\ 3 & -2 & 1 & -2 \end{bmatrix} \text{ in RREF } \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & -5 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

2. (i) If M is a 4×7 matrix, what is the largest dimension of the column space? What are the bounds of the dimension of the null space?
   (ii) If M is a 6×10 matrix that is onto, what is the dimension of the null space?
   (iii) If M is a 10×6 matrix that is 1-to-1, what is the dimension of the column space?
   (iv) Suppose that $A\vec{x} = \vec{b}$ is a system of linear equations where A is a 7×4 matrix whose null space has dimension zero. How many solutions can this system have? What is the dimension of the column space of A?

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1. A basis for $col(M)$ is $\left\{\begin{bmatrix}1\\0\\3\end{bmatrix}, \begin{bmatrix}0\\1\\-2\end{bmatrix}\right\}$ and $\dim(col(M)) = 2$.

A basis for $row(M)$ is $\{[1\ 0\ -3\ 2], [0\ 1\ -5\ 4]\}$ and $\dim(row(M)) = 2$.

$M\vec{x} = \vec{0}$ has solution $\begin{bmatrix}a\\b\\c\\d\end{bmatrix} = \begin{bmatrix}3c-2d\\5c-4d\\c\\d\end{bmatrix} = c\begin{bmatrix}3\\5\\1\\0\end{bmatrix} + d\begin{bmatrix}-2\\-4\\0\\1\end{bmatrix}$

A basis for $nul(M)$ is $\left\{\begin{bmatrix}3\\5\\1\\0\end{bmatrix}, \begin{bmatrix}-2\\-4\\0\\1\end{bmatrix}\right\}$ and $\dim(nul(M)) = 2$.

(2i) $M: \mathbb{R}^7 \to \mathbb{R}^4$, $0 \le \dim(col(M)) \le 4$ and

$0 \le \dim(\text{domain}) - \dim(nul(M)) \le 4$
$-7 \le -\dim(nul(M)) \le -3$
$\Rightarrow 3 \le \dim(nul(M)) \le 7$

(2ii) $M: \mathbb{R}^{10} \to \mathbb{R}^6$ with $\dim(col(M)) = 6 \Rightarrow \dim(nul(M)) = 4$.

(2iii) $M: \mathbb{R}^6 \to \mathbb{R}^{10}$ with $\dim(nul(M)) = 0 \Rightarrow \dim(col(M)) = 6$.

(2iv) $A: \mathbb{R}^4 \to \mathbb{R}^7$ with $\dim(nul(A)) = 0$
$\Rightarrow \dim(col(A)) = 4$.

If this system is consistent, as there are no free variables, there is one solution.