

Homework 12 MATH 304 Section 3

Solution

Assigned: Friday, October 17.
Potentially Collected: Friday, October 24.

1. For each of the following matrices, find a basis for the Null Space, Row Space, and the Column Space.

(a) $A = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & -5 & 4 \\ 3 & -2 & 1 & -2 \end{bmatrix}$ in RREF $\begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & -5 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(b) $B = \begin{bmatrix} 1 & 0 & -5 & 1 & 4 \\ -2 & 1 & 6 & -2 & -2 \\ 0 & 2 & -8 & 1 & 9 \end{bmatrix}$ in RREF $\begin{bmatrix} 1 & 0 & -5 & 0 & 7 \\ 0 & 1 & -4 & 0 & 6 \\ 0 & 0 & 0 & 1 & -3 \end{bmatrix}$

(c) $C = \begin{bmatrix} -2 & 4 & -2 & -4 \\ 2 & -6 & -3 & 1 \\ -3 & 8 & 2 & -3 \end{bmatrix}$ in RREF $\begin{bmatrix} 1 & 0 & 6 & 5 \\ 0 & 1 & 5/2 & 3/2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(d) $D = \begin{bmatrix} 1 & 2 & -5 & 11 & -3 \\ 2 & 4 & -5 & 15 & 2 \\ 1 & 2 & 0 & 4 & 5 \\ 3 & 6 & -5 & 19 & -2 \end{bmatrix}$ in RREF $\begin{bmatrix} 1 & 2 & 0 & 4 & 0 \\ 0 & 0 & 1 & -7/5 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(a) A basis for $\text{col}(A)$ is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} \right\}$

A basis for $\text{row}(A)$ is $\{ [1 \ 0 \ -3 \ 2], [0 \ 1 \ -5 \ 4] \}$

A basis for $\text{nul}(A)$ is $\left\{ \begin{bmatrix} 3 \\ 5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -4 \\ 0 \\ 1 \end{bmatrix} \right\}$

Solutions to $A\vec{x} = \vec{0}$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 3c - 2d \\ 5c - 4d \\ c \\ d \end{bmatrix}$$

$$= c \begin{bmatrix} 3 \\ 5 \\ 1 \\ 0 \end{bmatrix} + d \begin{bmatrix} -2 \\ -4 \\ 0 \\ 1 \end{bmatrix}$$

(b) A basis for $\text{col}(B)$ is $\left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix} \right\}$

A basis for $\text{row}(B)$ is $\{ [1 \ 0 \ -5 \ 1 \ 4], [-2 \ 1 \ 6 \ -2 \ -2], [0 \ 2 \ -8 \ 1 \ 9] \}$

Solutions to $B\vec{x} = \vec{0}$

$$\begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} 5c - 7e \\ 4c - 6e \\ c \\ 3e \\ e \end{bmatrix} = c \begin{bmatrix} 5 \\ 4 \\ 1 \\ 0 \\ 0 \end{bmatrix} + e \begin{bmatrix} -7 \\ -6 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

A basis for $\text{nul}(B)$ is

$$\left\{ \begin{bmatrix} 5 \\ 4 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ -6 \\ 0 \\ 3 \\ 1 \end{bmatrix} \right\}$$

(c) A basis for $\text{col}(C)$ is $\left\{ \begin{bmatrix} -2 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \\ 8 \end{bmatrix} \right\}$

A basis for $\text{row}(C)$ is $\{[-2 \ 4 \ -2 \ -4], [2 \ -6 \ -3 \ 1]\}$

$$\text{Solutions to } C\vec{x} = \vec{0} \quad \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -6c - 5d \\ -5/2c - 3/2d \\ c \\ d \end{bmatrix} = c \begin{bmatrix} -6 \\ -5/2 \\ 1 \\ 0 \end{bmatrix} + d \begin{bmatrix} -5 \\ -3/2 \\ 0 \\ 1 \end{bmatrix}$$

A basis for $\text{nul}(C)$ is $\left\{ \begin{bmatrix} -6 \\ -5/2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ -3/2 \\ 0 \\ 1 \end{bmatrix} \right\}$

(d) A basis for $\text{col}(D)$ is $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -5 \\ -5 \\ 0 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 5 \\ -2 \end{bmatrix} \right\}$

A basis for $\text{row}(D)$ is $\left\{ [1 \ 2 \ -5 \ 11 \ -3], [2 \ 4 \ -5 \ 15 \ 2], [1 \ 2 \ 0 \ 4 \ 5] \right\}$

$$\text{Solutions to } D\vec{x} = \vec{0} \quad \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} -2b - 4d \\ b \\ 7/5d \\ d \\ 0 \end{bmatrix} = b \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + d \begin{bmatrix} -4 \\ 0 \\ 7/5 \\ 1 \\ 0 \end{bmatrix}$$

A basis for $\text{nul}(D)$ is $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 7/5 \\ 1 \\ 0 \end{bmatrix} \right\}$