

Homework 26 MATH 304 Section 3

Assigned: Wednesday, December 3.
Potentially Collected: Wednesday, December 10.

1. Let $\vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} -4 \\ -5 \end{bmatrix}$, $\vec{a} = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} -1 \\ -3 \\ -4 \end{bmatrix}$, and $\vec{c} = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$.

- (a) Find $\|\vec{a}\|$, $\|\vec{b}\|$, and $\|\vec{c}\|$.
- (b) Find $\|\vec{u} - \vec{v}\|$ and $\|\vec{w} - \vec{v}\|$.
- (c) Find $\text{proj}_{\vec{a}}(\vec{c})$ and $\text{proj}_{\vec{b}}(\vec{c})$.
- (d) Find $\text{proj}_{\vec{v}}(\vec{u})$ and $\text{proj}_{\vec{v}}(\vec{w})$.
- (e) Find $\vec{v} \cdot \vec{w}$ and $\vec{a} \cdot \vec{b}$.

$$\left. \begin{array}{l} \vec{b} \cdot \vec{c} = 6 - 4 - 2 = 0 \text{ orthogonal} \\ \vec{b} \cdot \vec{d} = 3 + 2 = 5 \\ \vec{b} \cdot \vec{u} = -3/2 - 1/2 = -2 \\ \vec{c} \cdot \vec{d} = 2 - 1 = 1 \end{array} \right\} \begin{array}{l} \vec{c} \cdot \vec{u} = -1 + 1/4 = -3/4 \\ \vec{d} \cdot \vec{u} = -1/2 - 1/4 = -3/4 \end{array}$$

2. Which of the following vectors are orthogonal? in the same direction? in opposing directions?

$$\vec{a} = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} \quad \vec{c} = \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix} \quad \vec{d} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad \vec{u} = \begin{bmatrix} -1/2 \\ 0 \\ -1/4 \end{bmatrix}$$

3. Let $\vec{w} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ and $\vec{x} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$. Find all the vectors \vec{v} where $\vec{v} \perp \vec{w}$ and $\vec{v} \perp \vec{x}$.

② $\vec{a} \cdot \vec{b} = 3 + 1 - 4 = 0$ orthogonal
 $\vec{a} \cdot \vec{c} = 2 - 4 + 2 = 0$ orthogonal
 $\vec{a} \cdot \vec{d} = 1 + 0 - 2 = -1$
 $\vec{a} \cdot \vec{u} = -1/2 + 1/2 = 0$ orthogonal

$$\begin{array}{l} \vec{a} \perp \vec{b}, \vec{a} \perp \vec{c} \\ \vec{a} \perp \vec{u}, \vec{b} \perp \vec{c} \end{array}$$

\vec{x} is in the same or opposite direction as \vec{y} if there is a nonzero scalar c where $c\vec{x} = \vec{y}$.