Homework 11 MATH 304 Section 3

Solution

Assigned: Wednesday, October 15.
Potentially Collected: Wednesday, October 22.

1. Which of the following vectors in $\mathbb{P}_2$, the vector space of degree 2 polynomials, are linearly dependent? For those which are, express one vector as a linear combination of the rest.

   (i) $\{x^2+1, x-2, x+3\}$.

   (ii) $\{2x^2+x, x^2+3, x\}$.

   (iii) $\{2x^2+x+1, 3x^2+x-5, x+13\}$.

   $x+13 = 3(2x^2+x+1) - 2(3x^2+x-5)$

2. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by the matrix $A = \begin{bmatrix} -1 & 2 \\ 4 & -8 \end{bmatrix}$.

   Solve the matrix $\left[\begin{array}{cc|ccc} -1 & 2 & -4 & 0 & 12 \\ 4 & -8 & 1 & 5 & -3 \end{array}\right]$

   RREF $\left[\begin{array}{cc|ccc} 1 & -2 & 0 & 4/3 & 0 \\ 0 & 0 & 1 & -1/3 & -3 \end{array}\right]$

   (i) Which of the following are in the image of T?

   NONE

   (a) $\begin{bmatrix} -4 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 5 \end{bmatrix}$ (c) $\begin{bmatrix} 12 \\ -3 \end{bmatrix}$

   (ii) Which of the following are in the kernel of T? $\begin{bmatrix} -1 & 2 \\ 4 & -8 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 8 \end{bmatrix} + \begin{bmatrix} 6 \\ -24 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

   $\begin{bmatrix} -1 & 2 \\ 4 & -8 \end{bmatrix}\begin{bmatrix} 10 \\ 5 \end{bmatrix} = \begin{bmatrix} -10 \\ 40 \end{bmatrix} + \begin{bmatrix} 10 \\ -40 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

   ✓ (a) $\begin{bmatrix} 10 \\ 5 \end{bmatrix}$ ~~(b)~~ $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ ~~(c)~~ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

   $\begin{bmatrix} -1 & 2 \\ 4 & -8 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix} + \begin{bmatrix} 2 \\ -8 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

3. Determine which of the following sets of vectors are bases for $\mathbb{R}^4$. If the set is not a basis, then say which conditions in the definition of a basis fails.

   $$S = \left\{ \begin{bmatrix} 1 \\ -3 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ 6 \\ 4 \\ -8 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \\ -4 \end{bmatrix} \right\}$$

   RREF $\begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Not a basis, LD, doesn't span $\mathbb{R}^4$

   $$T = \left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \\ -2 \end{bmatrix} \right\}$$

   RREF $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ A basis for $\mathbb{R}^4$ as T is invertible.

1i $0 = a(x^2+1) + b(x-2) + c(x+3)$
$= ax^2 + (b+c)x + (a-2b+3c)$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & -2 & 3 & 0 \end{array}\right]$ in RREF

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \Rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

LI

1ii $0 = a(2x^2+x) + b(x^2+3) + c(x)$
$= (2a+b)x^2 + (a+c)x + 3b$

$\left[\begin{array}{ccc|c} 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 3 & 0 & 0 \end{array}\right]$ in RREF $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$ LI

1iii $0 = a(2x^2+x+1) + b(3x^2+x-5) + c(x+13)$

$\left[\begin{array}{ccc|c} 2 & 3 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & -5 & 13 & 0 \end{array}\right]$ in RREF $\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ LD