

# Homework 25 MATH 304 Section 3

**Assigned:** Wednesday, November 26.  
**Potentially Collected:** Wednesday, December 3.

1. Which of the following matrices are diagonalizable?

(a)  $\begin{bmatrix} 1 & 4 \\ 1 & -2 \end{bmatrix}$

(b)  $\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

(c)  $\begin{bmatrix} 1 & 1 & -2 \\ 4 & 0 & 4 \\ 1 & -1 & 4 \end{bmatrix}$

2. Let  $A = \begin{bmatrix} 3 & -5 \\ 1 & -3 \end{bmatrix}$ . Compute  $A^9$  by finding a matrix  $P$  such that  $P^{-1}AP$  is a diagonal matrix  $D$  and show that  $A^9 = PD^9P^{-1}$ .

$$p_A(\lambda) = \det \left( \begin{bmatrix} 3-\lambda & -5 \\ 1 & -3-\lambda \end{bmatrix} \right) = (3-\lambda)(-3-\lambda) + 5$$

$$= -9 + \lambda^2 + 5 = (\lambda - 2)(\lambda + 2)$$

Eigenvalues:  $\lambda \in \{-2, 2\}$

$$E_{-2} = \text{nul} \left( \begin{bmatrix} 5 & -5 \\ 1 & -1 \end{bmatrix} \right) = \text{span} \left( \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right)$$

$$E_2 = \text{nul} \left( \begin{bmatrix} 1 & -5 \\ 1 & -5 \end{bmatrix} \right) = \text{span} \left( \begin{bmatrix} 5 \\ 1 \end{bmatrix} \right)$$

$$X = \left( \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \end{bmatrix} \right)$$

$$S = (\vec{e}_1, \vec{e}_2)$$

$$P = {}_S I_X = \begin{bmatrix} 1 & 5 \\ 1 & 1 \end{bmatrix}$$

$$P^{-1} = {}_X I_S = \frac{1}{4} \begin{bmatrix} -1 & 5 \\ 1 & -1 \end{bmatrix}$$

$$D = P^{-1}AP = \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$D^9 = \begin{bmatrix} -2^9 & 0 \\ 0 & 2^9 \end{bmatrix} = \begin{bmatrix} -512 & 0 \\ 0 & 512 \end{bmatrix}$$

$$D^9 = (P^{-1}AP)^9 = P^{-1}AP \dots P^{-1}AP$$

$$= P^{-1}A^9P \Rightarrow A^9 = PD^9P^{-1}$$

$$A^9 = \begin{bmatrix} 1 & 5 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -128 & 0 \\ 0 & 128 \end{bmatrix} \begin{bmatrix} -1 & 5 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 5 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 128 & -640 \\ 128 & -128 \end{bmatrix} = \begin{bmatrix} 768 & -1280 \\ 256 & -768 \end{bmatrix}$$