Homework 24 MATH 304 Section 3

Assigned: Monday, November 21.
Potentially Collected: Wednesday, December 3.

1. Find the characteristic polynomials, the eigenvalues, and associated eigenvectors for each of the following matrices.

   (a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$

   (c) $\begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

   (d) $\begin{bmatrix} 2 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & -2 & 1 \end{bmatrix}$

$P_A(\lambda) = \det\left(\begin{bmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{bmatrix}\right) = (1-\lambda)^2 - 1 = \lambda^2 - 2\lambda = \lambda(\lambda-2)$

Eigenvalues: $\lambda \in \{0, 2\}$

$E_0 = \text{null}\left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\right) = \text{Span}\left(\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right)$ $E_2 = \text{nul}\left(\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\right) = \text{Span}\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)$

$P_B(\lambda) = \det\left(\begin{bmatrix} 1-\lambda & -1 \\ 2 & 4-\lambda \end{bmatrix}\right) = (1-\lambda)(4-\lambda) + 2 = \lambda^2 - 5\lambda + 6$
$= (\lambda-3)(\lambda-2)$

Eigenvalues: $\lambda \in \{2, 3\}$

$E_2 = \text{nul}\left(\begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix}\right) = \text{Span}\left(\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right)$ $E_3 = \text{nul}\left(\begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix}\right) = \text{Span}\left(\begin{bmatrix} -\frac{1}{2} \\ 1 \end{bmatrix}\right)$

$P_C(\lambda) = \det\left(\begin{bmatrix} -\lambda & 1 & 2 \\ 0 & -\lambda & 3 \\ 0 & 0 & -\lambda \end{bmatrix}\right)$
$= -\lambda^3$

Eigenvalues: $\lambda \in \{0\}$ $E_0 = \text{nul}\left(\begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}\right) = \text{Span}\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right)$

$P_D(\lambda) = \det\left(\begin{bmatrix} 2-\lambda & 2 & 3 \\ 1 & 2-\lambda & 1 \\ 2 & -2 & 1-\lambda \end{bmatrix}\right) = (2-\lambda)((2-\lambda)(1-\lambda)+2) - 2(1-\lambda-2) + 3(-2-2(2-\lambda))$
$= -\lambda^3 + 5\lambda^2 - 2\lambda - 8 = (\lambda-2)(-\lambda^2+3\lambda+4)$
$= -(\lambda-2)(\lambda-4)(\lambda+1)$

Eigenvalue: $\lambda \in \{-1, 2, 4\}$

$E_{-1} = \text{nul}\left(\begin{bmatrix} 3 & 2 & 3 \\ 1 & 3 & 1 \\ 2 & -2 & 2 \end{bmatrix}\right) = \text{Span}\left(\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}\right)$

$E_2 = \text{nul}\left(\begin{bmatrix} 0 & 2 & 3 \\ 1 & 0 & 1 \\ 2 & -2 & -1 \end{bmatrix}\right) = \text{Span}\left(\begin{bmatrix} -1 \\ -1.5 \\ 1 \end{bmatrix}\right)$

$E_4 = \text{nul}\left(\begin{bmatrix} -2 & 2 & 3 \\ 1 & -2 & 1 \\ 2 & -2 & -3 \end{bmatrix}\right) = \text{Span}\left(\begin{bmatrix} 4 \\ 2.5 \\ 1 \end{bmatrix}\right)$