Homework 20 MATH 304 Section 3

Assigned: Friday, November 14.
Potentially Collected: Friday, November 21.

1. Let V be the vector space with basis $S = (1, t, e^t, te^t)$ and note that V is a subspace of the vector space of differentiable real functions. Let $L : V \to V$ be the linear transformation where $L(f(t)) = f'(t)$ for any differentiable function f. Find ${}_SL_S$.

2. Let $F : \mathbb{P}_1 \to \mathbb{P}_2$ be the linear transformation where $F(p(x)) = xp(x) + p(0)$ for any degree one polynomial p. Consider the bases $S = (x, 1)$ and $R = (x + 1, x - 1)$ for $\mathbb{P}_2$ and $A = (x^2, 1, x)$ and $B = (x^2 + 1, x - 1, x + 1)$ for $\mathbb{P}_3$.

   (a) Find the standard matrix for F and then calculate ${}_AF_S$ and ${}_BF_R$ using change of basis matrices.

   (b) Calculate $F(-3x - 3)$ for each of the three matrices of (a).

3. Let $L : M_{22} \to M_{22}$ be the linear transformation between the vector space of 2×2 real matrices where $L(A) = \left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} A\right)$ for any 2×2 real matrix A. Let

$$S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

$$T = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}$$

   (a) Show that S and T are bases for M_{22}.

   (b) Find ${}_SL_S$, ${}_TL_T$, ${}_SL_T$, and ${}_TL_S$.