Homework 10 MATH 304 Section 3

<u>Solution</u>

Assigned: Monday, October 13.
Potentially Collected: Monday, October 20.

1. Which of the following vectors in $\mathbb{R}^3$ are linearly dependent? For those which are, express one vector as a linear combination of the rest.

   (i) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 6 \end{bmatrix} \right\}.$

   in RREF $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$ $\begin{bmatrix} 3 \\ 6 \\ 6 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + 1\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix} + 1\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

   LD

   (ii) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix} \right\}.$

   in RREF $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ LI

   (iii) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}.$

2. For what values of c are the vectors $\vec{v_1} = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$, $\vec{v_2} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$, and $\vec{v_3} = \begin{bmatrix} 1 \\ 1 \\ c \end{bmatrix}$ linearly dependent?

3. For what values of c are the vectors $x + 3$ and $2x + (c^2 + 2)$ in P_1, the vector space of degree 1 polynomials, linearly independent?

in RREF $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$ $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 0\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 0\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

LD

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② To be LD there must be a nontrivial solution to $\left[\begin{array}{ccc|c} -1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ -1 & 2 & c & 0 \end{array}\right]$

$\begin{bmatrix} -1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 2 & c \end{bmatrix} \xrightarrow{r_3 = R_3 - R_1} \begin{bmatrix} -1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & c-1 \end{bmatrix}$ there will be a free variable when $c - 1 = 0$

$\Rightarrow \boxed{c = 1}$

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③ To be LD there must be a nontrivial solution to

$0 = a(x+3) + b(2x + (c^2+2)) \Rightarrow 0 = (a+2b)x + (3a + bc^2 + 2b)$

$\Rightarrow a + b = 0$
$3a + (c^2+2)b = 0$ $\Rightarrow \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 3 & c^2+2 & 0 \end{array}\right] \xrightarrow{r_2 = R_2 - 3R_1} \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & c^2-1 & 0 \end{array}\right]$ To be nontrivial $c^2 - 1 = 0 \Rightarrow c = \pm 1$

$\boxed{\text{So to be LI, } c \neq \pm 1}$