

Homework 11 MATH 304 Section 3

Assigned: Wednesday, October 15.

Potentially Collected: Wednesday, October 22.

1. Which of the following vectors in \mathbb{P}_2 , the vector space of degree 2 polynomials, are linearly dependent? For those which are, express one vector as a linear combination of the rest.

(i) $\{x^2 + 1, x - 2, x + 3\}$.

(ii) $\{2x^2 + x, x^2 + 3, x\}$.

(iii) $\{2x^2 + x + 1, 3x^2 + x - 5, x + 13\}$.

2. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be defined by the matrix $A = \begin{bmatrix} -1 & 2 \\ 4 & -8 \end{bmatrix}$.

- (i) Which of the following are in the image of T ?

(a) $\begin{bmatrix} -4 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 5 \end{bmatrix}$ (c) $\begin{bmatrix} 12 \\ -3 \end{bmatrix}$

- (ii) Which of the following are in the kernel of T ?

(a) $\begin{bmatrix} 10 \\ 5 \end{bmatrix}$ (b) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

3. Determine which of the following sets of vectors are bases for \mathbb{R}^4 . If the set is not a basis, then say which conditions in the definition of a basis fails.

$$S = \left\{ \begin{bmatrix} 1 \\ -3 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ 6 \\ 4 \\ -8 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \\ -4 \end{bmatrix} \right\}$$

$$T = \left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \\ -2 \end{bmatrix} \right\}$$