Bonus: $A = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ where $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 3 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 4 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 4 \\ -2 \\ 0 \\ -2 \end{bmatrix}$, $\vec{v}_4 = \begin{bmatrix} 6 \\ 3 \\ 0 \\ 8 \end{bmatrix}$

① If $\vec{x} = \begin{bmatrix} -1 \\ 7 \\ 5 \\ 1 \end{bmatrix}$, $\vec{y} = \begin{bmatrix} 6 \\ -1 \\ -8 \\ -8 \end{bmatrix}$

which of $\vec{x}$ or $\vec{y}$ are in Span (A)?

② For the vector in Span(A) find $\{a_1, a_2, a_3, a_4\}$ where the vector equals $a_1\vec{v}_1 + a_2\vec{v}_2 + a_3\vec{v}_3 + a_4\vec{v}_4$.

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① if $\vec{z} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ is in span(A) then $\left[\begin{array}{cccc|c} 1 & 0 & 4 & 6 & a \\ 2 & 1 & -2 & 3 & b \\ -1 & 2 & 0 & 0 & c \\ 3 & 4 & -2 & 8 & d \end{array}\right]$ is consistent.

$r_2 = R_2 - 2R_1$
$r_3 = R_3 + R_1$
$r_4 = R_4 - 3R_1$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4 & 6 & a \\ 0 & 1 & -10 & -9 & b-2a \\ 0 & 2 & 4 & 6 & c+a \\ 0 & 4 & -14 & -10 & d-3a \end{array}\right]$$

$r_3 = R_3 - 2R_2$
$r_4 = R_4 - 4R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4 & 6 & a \\ 0 & 1 & -10 & -9 & b-2a \\ 0 & 0 & 24 & 24 & c-2b+5a \\ 0 & 0 & 26 & 26 & d-4b+5a \end{array}\right]$$

$r_3 = \frac{1}{24}R_3$, $r_4 = \frac{1}{26}R_4$ THEN $r_4 = R_4 - R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4 & 6 & a \\ 0 & 1 & -10 & -9 & b-2a \\ 0 & 0 & 1 & 1 & (c-2b+5a)/24 \\ 0 & 0 & 0 & 0 & \frac{d-4b+5a}{26} - \frac{c-2b+5a}{24} \end{array}\right]$$

to be consistent this expression MUST be zero:

$\vec{x}$: $\frac{(1)-4(7)+5(-1)}{26} - \frac{(5)-2(7)+5(-1)}{24}$

$\vec{y}$: $\frac{(-8)-4(-1)+5(6)}{26} - \frac{(-8)-2(-1)+5(6)}{24}$

$\vec{x} \notin$ Span(A) and $\vec{y} \in$ Span(A)

② So $\vec{y} \in \text{Span}(A)$, let $\vec{y}$ replace $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4 & 6 & 6 \\ 0 & 1 & -10 & -9 & -13 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} r_1 = R_1 - 4R_3 \\ r_2 = R_2 + 10R_3 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 2 \\ 0 & 1 & 0 & 1 & -3 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

now $\vec{y} = a_1\vec{v}_1 + a_2\vec{v}_2 + a_3\vec{v}_3 + a_4\vec{v}_4$

where
$$\left.\begin{array}{l} a_1 = 2 - 2a_4 \\ a_2 = -3 - a_4 \\ a_3 = 1 - a_4 \\ a_4 \text{ anything} \end{array}\right\} \xrightarrow{\text{Solutions of the form}} \begin{bmatrix} 2-2a_4 \\ -3-a_4 \\ 1-a_4 \\ a_4 \end{bmatrix}$$

which can be written
$$\begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix} + a_4 \begin{bmatrix} -2 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$