

Find $\det(A)$ and $\det(A^3)$ when

Homework Bonus
11/20

$$A = \begin{bmatrix} 1 & 2 & -3 & 4 \\ -4 & 2 & 1 & 3 \\ 3 & 0 & 0 & -3 \\ 2 & 0 & -2 & 3 \end{bmatrix}$$

Note: $\det(A^3) = (\det(A))^3$

Cofactor Expansion

expand column 2

$$\det(A) = 2(-1)^{1+2} \det \begin{pmatrix} -4 & 1 & 3 \\ 3 & 0 & -3 \\ 2 & -2 & 3 \end{pmatrix} + 2(-1)^{2+2} \det \begin{pmatrix} 1 & -3 & 4 \\ 3 & 0 & -3 \\ 2 & -2 & 3 \end{pmatrix}$$

$$= -2 \left(1(-1)^{1+2} \det \begin{pmatrix} 3 & -3 \\ 2 & 3 \end{pmatrix} + (-2)(-1)^{3+2} \det \begin{pmatrix} -4 & 3 \\ 3 & -3 \end{pmatrix} \right) \text{ expand column 2}$$

$$+ 2 \left(3(-1)^{2+1} \det \begin{pmatrix} -3 & 4 \\ -2 & 3 \end{pmatrix} + (-3)(-1)^{2+3} \det \begin{pmatrix} 1 & -3 \\ 2 & -2 \end{pmatrix} \right) \text{ expand row 2}$$

$$= -2 \left(-(9+6) + 2(12-9) \right) + 2 \left(-3(-9+8) + 3(-2+6) \right)$$

$$= -2(-15+6) + 2(3+12) = -2(-9) + 2(15) = \boxed{48}$$

$$\det(A^3) = 48^3$$