$F: \mathbb{P}_2 \to \mathbb{P}_1$ where $F(p(x)) = p'(1)x + p(1)$

$A = (1+x, -1)$ $\qquad$ $E_1 = (1, x)$

$B = (1-3x^2, x, 1+x)$ $\qquad$ $E_2 = (1, x, x^2)$

Homework Bonus
11/13/2014

① Find ${}_AF_B$.

② Use it to find $F(x^2+1)$.

$${}_AF_B = [K_A F(1-3x^2) \;\; K_A F(x) \;\; K_A F(1+x)]$$

$$= [K_A(-6x-2) \;\; K_A(x+1) \;\; K_A(x+2)]$$

now solve for [redacted]

$$\left[\begin{array}{cc|ccc} 1 & -1 & -2 & 1 & 2 \\ 1 & 0 & -6 & 1 & 1 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{cc|ccc} 1 & 0 & -6 & 1 & 1 \\ 0 & 1 & -4 & 0 & -1 \end{array}\right]$$

$$= \begin{bmatrix} -6 & 1 & 1 \\ -4 & 0 & -1 \end{bmatrix}$$

Alternatively, ${}_AF_B = ({}_AI_{E_1})({}_{E_1}F_{E_2})({}_{E_2}I_B)$

$${}_AI_{E_1} = [K_A(1) \;\; K_A(x)] = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$$

$${}_{E_2}I_B = [K_{E_2}(1-3x^2) \;\; K_{E_2}(x) \;\; K_{E_2}(1+x)] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ -3 & 0 & 0 \end{bmatrix}$$

$${}_{E_1}F_{E_2} = [K_{E_1}F(1) \;\; K_{E_1}F(x) \;\; K_{E_1}F(x^2)] = [K_{E_1}(1) \;\; K_{E_1}(x+1) \;\; K_{E_1}(2x+1)]$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$${}_AF_B = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ -3 & 0 & 0 \end{bmatrix}$$

$$F(x^2+1) = K_A^{-1}(_AF_B)\underbrace{K_B(x^2+1)}$$

$\llcorner$ solve $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ -3 & 0 & 0 & 1 \end{array}\right] \overset{RREF}{=} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1/3 \\ 0 & 1 & 0 & -4/3 \\ 0 & 0 & 1 & 4/3 \end{array}\right]$

$$= K_A^{-1}\left(\begin{bmatrix} -6 & 1 & 1 \\ -4 & 0 & -1 \end{bmatrix}\begin{bmatrix} -1/3 \\ -4/3 \\ 4/3 \end{bmatrix}\right)$$

$$= K_A^{-1}\left(\begin{bmatrix} 2 \\ 0 \end{bmatrix}\right) = 2(x+1) + 0(-1) = 2x+2$$

<u>Check</u> $F(x^2+1) = (2(1))x + (1^2+1) = 2x+2$