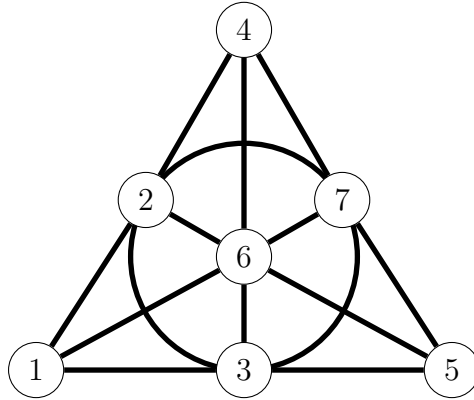


Quiz 7  
May 4th, 2017



- 1) Write down the blocks corresponding to the  $(7, 3, 1)$ -design given by the above labeling of the Fano plane.
- 2) Let  $D$  be a design on  $[v]$ . The *Complimentary Design* of  $D$  is formed by taking the complements of all the blocks. That is, if  $D$  has blocks  $\mathcal{B} = \{B_1, B_2, \dots, B_b\}$ , the complimentary design has blocks

$$\mathcal{B}' = \{[v] \setminus B_1, [v] \setminus B_2, \dots, [v] \setminus B_b\}.$$

- a) Write down the blocks of the complimentary design to the design from problem 1).
- b) If a design has parameters  $(v, b, r, k, \lambda)$ , what are the parameters of its complimentary design?
- 3) Let  $D$  be a symmetric design on  $[v]$ , and let  $B_i$  be one of its blocks. The *Residual Design of  $D$  on  $[v] \setminus B_i$*  is formed by taking the points of  $B_i$  out of all the blocks, and then deleting (the now empty)  $B_i$ . For example, if  $D$  has blocks  $\mathcal{B} = \{B_1, B_2, B_3, \dots, B_b\}$ , the residual design on  $[v] \setminus B_2$  has blocks

$$\mathcal{B}' = \{B_1 \setminus B_2, B_3 \setminus B_2, \dots, B_b \setminus B_2\}.$$

- a) Write down the blocks of the design from problem 1) which is residual on  $[7] \setminus \{3, 4, 6\}$ .
- b) If a design has parameters  $(v, b, r, k, \lambda)$ , what are the parameters of the residual design on  $[v] \setminus$  any block?
- c) If we take an affine plane, add the points and line at infinity necessary to make it a projective plane, write down the lines as the blocks of a design, and then take the residual design on the block corresponding to the line at infinity, what do we get?
- 4) Let  $D$  be a symmetric design on  $[v]$ , and let  $B_i$  be one of its blocks. The *Derived Design* of  $D$  with respect to  $B_i$  is formed by taking the intersections of all the blocks with  $B_i$ , and then deleting  $B_i$ . For example, if  $D$  has blocks  $\mathcal{B} = \{B_1, B_2, B_3, \dots, B_b\}$ , the derived design on  $B_2$  has blocks

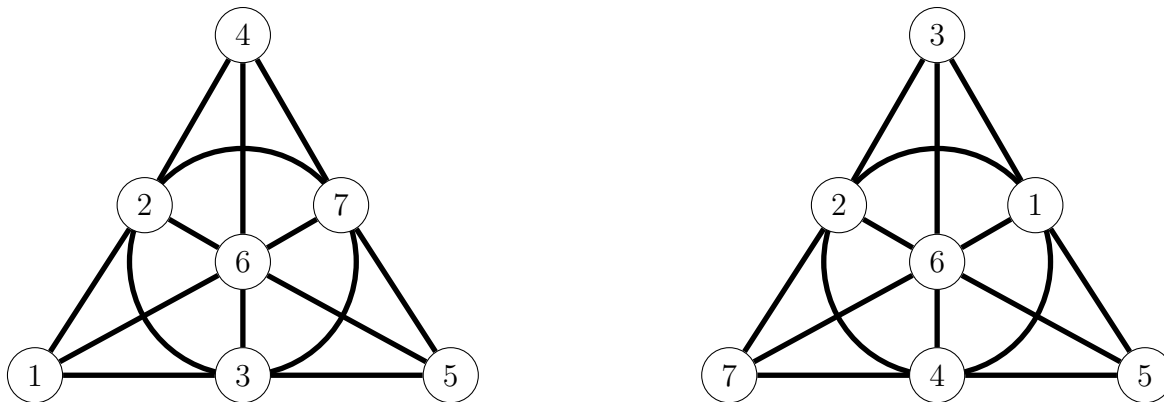
$$\mathcal{B}' = \{B_2 \cap B_1, B_2 \cap B_3, \dots, B_2 \cap B_b\}.$$

- a) Write down the blocks of the design from problem 1), derived with respect to  $\{3, 4, 6\}$ .
- b) If a design has parameters  $(v, b, r, k, \lambda)$ , what are the parameters of the derived design on any block?

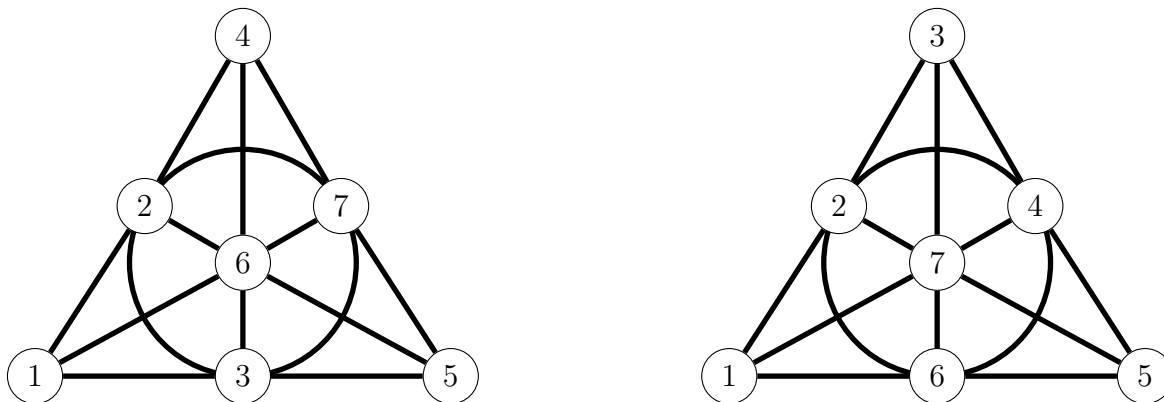
- 5) Let  $D$  be a design on  $[v]$ , with no repeated blocks. The *Subtract Design* of  $D$  is formed by taking as blocks all of the subsets of  $[v]$  of size  $k$  which are not elements of  $\mathcal{B}$ . That is, if  $D$  has blocks  $\mathcal{B} = \{B_1, B_2, B_3, \dots, B_b\}$ , the subtract design has blocks

$$\mathcal{B}' = \binom{[v]}{k} \setminus \{B_1, B_2, B_3, \dots, B_b\}.$$

- a) Write down the blocks of the subtract design for the design from problem 1).  
 b) If a design has parameters  $(v, b, r, k, \lambda)$ , what are the parameters of its subtract design?
- 6) Of the  $7!$  ways to label the points in a Fano plane, some labelings will give the same list of blocks if we write down the associated design, while some will not. For example,



These two labelings give the same 7 blocks,



while these do not. Call two labelings *different* if they give different blocks. How many different labelings of the Fano plane are there? (HINT: for a given set of blocks, count how many labelings give that set.)

rules:

1. Work out the problems on scratch paper, and transfer your neat and clean solutions onto the blue book provided. Turn in only the blue book (make sure all your names are on it).
2. The quiz is open book and open notes, but cell phones may not be out.
3. Failure of any group member to follow the rules will result in a score of zero for every group member.