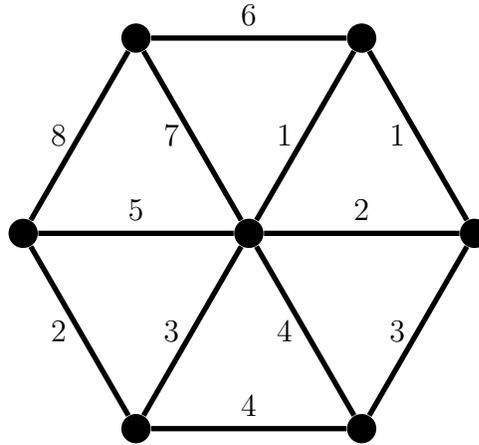


Quiz 4
March 24th, 2017

1) Let G be the edge-weighted graph below:



- Find a minimum-weight spanning tree of G , and give its weight.
- How many Hamilton cycles does G have?
- Find a minimum-weight Hamilton cycle of G , and give its weight.
- (BONUS) This kind of graph, formed by taking an n -cycle and placing an additional vertex in the middle, adjacent to all n exterior vertices, is called a *wheel* and denoted W_n . The central vertex is called the *hub*; the outer cycle is called the *rim*. Guess what the edges from the hub to the rim are called.

2) Recall that C_4 is a 4-cycle.

- Draw a copy of C_4 and assign the weights $\{1, 1, 2, 2\}$ to the edges in a way such that the minimum-weight spanning tree is unique.
- Draw a copy of C_4 and assign the weights $\{1, 1, 2, 2\}$ to the edges in a way such that the minimum-weight spanning tree is *not* unique.

- 3) SHORT ANSWER: Briefly explain, or draw an example or a counterexample (if applicable).
- a) I claim that a labeled K_n has $(n - 1)!$ Hamilton cycles (for large enough n): every Hamilton cycle goes through vertex 1, so write it down, then permute the other $n - 1$ vertices any way you want (the edges are always there, so this always gives a path through all the vertices, which I can end by going back to vertex 1). Am I right?
 - b) If a graph is Eulerian (has a closed Eulerian tour), is it necessarily Hamiltonian (has a Hamilton cycle)?
 - c) If a graph is Hamiltonian, is it necessarily Eulerian?
 - d) Draw a tree that is not bipartite.
 - e) Draw a bipartite graph that is not a tree.
 - f) Is every subgraph of a connected graph connected?
 - g) Is every *induced* subgraph of a connected graph connected?
 - h) Is every subgraph of a tree a forest? (A tree is a forest with one component, by the way)
 - i) How many spanning trees does a labeled C_n have?
 - j) Let $k \cdot C_n$ be the graph made by copying the edges in C_n k times each (for $k = 1$, this is C_n , for $k \geq 2$ the graph is not simple), and labeling everything. How many spanning trees does $k \cdot C_n$ have?

rules:

1. Work out the problems on scratch paper, and transfer your neat and clean solutions onto the blue book provided. Turn in only the blue book (make sure all your names are on it).
2. The quiz is open book and open notes, but cell phones may not be out.
3. Failure of any group member to follow the rules will result in a score of zero for every group member.