

Math 314: Discrete Mathematics

Homework 7

Submission instructions:

- a) This assignment is due Wednesday, April 19th at 8:00 AM.
- b) The rules for submissions given on the website still apply (you still need to copy the problems down, etc.). You do not need to print off and attach this sheet (but you can).

Questions:

1. Problem 10.1.2. (Your graphs must not be the ones in the back)
2.
 - a) Problem 10.4.10.
 - b) Surprise surprise, you've done this problem before too. Identify the problem, and *carefully* explain how the two are the same.
3. Problem 10.4.5.
4. If a bipartite graph has $k > 0$ connected components, each has a nonempty vertex set, all the vertices are labeled, and one component has at least one edge, how many "bipartitions" of the vertices are there? That is, how many ways are there of writing the vertex set as $A \cup B$ such that $A \cap B = \emptyset$ and the induced subgraphs on A and on B have no edges? Does your formula still work if I don't require that one component has at least one edge? Why or why not?
5. Problem 10.4.11.

Optional

6. (You don't have to copy this problem down if you don't want to) We are going to take problem 10.4.11 and put it on steroids. For any n , the *n-dimensional cube* is obtained by making a graph whose vertices are the subsets of $[n]$, where two vertices are adjacent if the subsets they correspond to have a symmetric difference with size exactly 1 (this is just another way of describing exactly the situation of 10.4.11).
 - a) Give the number of vertices and edges of this graph.
 - b) Prove that this graph is regular and give the degree of the vertices.

- c) Prove that this graph is bipartite (Hint: choose a vertex to put in one part, then look at the symmetric differences of its subset with all others to figure out where to put the rest), and then, using only this fact and part b), prove it has a perfect matching.
- d) Find the diameter of this graph.
- e) (Challenging) Prove that this graph is Hamiltonian (this can be done by induction, but it can also be done without it).