Math 314: Discrete Mathematics
Homework 2

Submission instructions:

a) Choose 5 of the first 9 problems according to the guidelines below. You may also do #10 and/or #11 if you wish. Do not turn in more than 5 of the first 9 problems!

b) Staple this page on top of your solutions (because it will be the top page, make sure your name is on it).

c) This assignment is due Wednesday, February 8th at 8:00 AM.

d) The rules for submissions given previously still apply (you still need to copy the problems down, etc.).

Questions:

Do ONE of questions 1 and 2.

1. a) Prove that if $x \notin \mathbb{Q}$, then $\sqrt{x} \notin \mathbb{Q}$.
   b) Prove that the converse is false.

2. You have an 8×8 chessboard and a large supply of dominoes which take up exactly 2 squares on the board. You are attempting to tile the board by laying down dominoes, which may not overlap and may not be placed at angles. Prove that, if two opposite corners of the board are removed, the resulting 62-square board cannot be tiled by dominoes.

Do ONE of questions 3 and 4.

3. While it is not always true that $n! < \left(\frac{n}{2}\right)^n$, it is true once n is large enough. Find the smallest such $n > 0$, and then use induction to prove that the inequality continues to hold. You may use the fact that

$$\left(\frac{n+1}{n}\right)^n \geq 2$$

   (as you might recall, it approaches e).

4. Problem 2.5.1.

Do ONE of questions 5 and 6.

5. Problem 1.8.29.

6. Problem 2.5.5.

Do TWO of questions 7 through 9.

7. The sum of five positive real numbers is 100. Prove that there are two numbers among them whose difference is at most 10.

8. Prove that at any party with $n \geq 2$ people, there are two people who know the same number of people at the party.

9. Problem 2.5.8.

Optional problems.

10. Prove that if you put 5 dots on an orange with a marker, there is a hemisphere containing 4 of them (assume a dot on your cut is in both hemispheres).

11. (challenging) Give an example of three infinite sets of natural numbers with the property that the intersection of any two of them is infinite, but the intersection of all three of them is empty.