

## Exam 2 Review

1) Recall that the *diameter* of a graph is the maximum distance between two vertices in the graph (*distance* being the number of edges in a shortest path between two vertices).

a) Find the diameter of  $K_n$ ,  $K_{m,n}$ , the cube,  $P_n$ , and  $C_n$  ( $m, n > 0$  to avoid weirdness).

b) Prove that if a graph has diameter  $d$ , then there are two vertices of distance  $c$  for all  $1 \leq c \leq d$ .

2) The *line graph*  $L(G)$  of a graph  $G = (V, E)$  is formed as follows:  $V(L(G)) = E(G)$ , and two vertices of  $L(G)$  are adjacent iff the corresponding edges of  $G$  are adjacent.

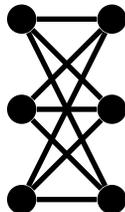
a) Draw the line graph of  $K_4$  and the Petersen graph.

b) Prove that  $C_n$  is its own line graph. (HINT: how many vertices does it have? Is it regular? Of what degree?)

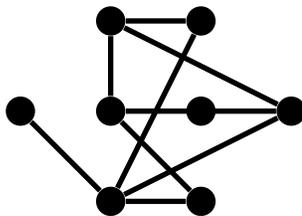
c) Prove the line graph of a regular graph is regular. (HINT: what degree must each vertex of  $L(G)$  have?)

3) Draw the labeled tree with Prüfer code 1 1 1 2 2 2 1 2 1

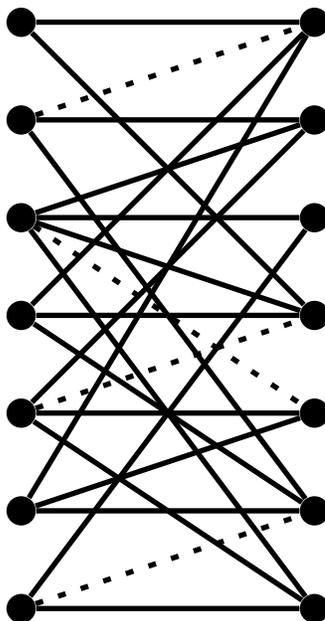
4) Is the following graph 4-colorable? Why or why not?



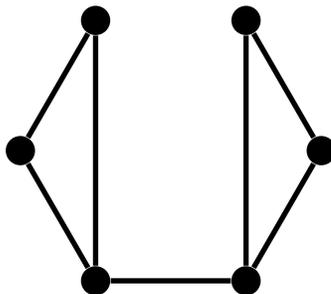
6) Write down every graph property you can possessed by the following graph.



- 7) Let  $T$  be a tree in which every leaf is adjacent to a vertex of degree at least 3. Prove there are two leaves with a common neighbor. (HINT: what happens when you delete a leaf from a tree?)
- 8) Use the augmenting path algorithm to extend the indicated maximal matching to a perfect matching.



9) Assign the weights  $\{1, 2, 2, 2, 3, 3, 3\}$  to the edges of the graph below in a way that every spanning tree has the same weight.



*notes:*

1. This review does not claim to be (nor is it) complete. Anything we have done in class, anything similar to the homework or quizzes, and anything similar to questions in the book is fair game.
2. The test will cover all of chapters 7-10, 12, and 13. To be adequately prepared you should be ready to answer questions from any of that.
3. For additional practice, please refer to the homework assignments, the problems in the book, your notes, and the quizzes.