

MATH 304

Test 1

7/13/16

Name:

Solutions

INSTRUCTIONS

Before the test begins, you must put away all electronic devices and any paper or written material. Your cell phone must be turned off. When instructed, turn over this cover page and begin the exam. You will have 30 minutes to complete the exam.

Problem	1	2	3	4	TOTAL
Maximum Score	10	20	15	25	70
Your Score					

1. (10 points) Use elementary row operations to convert the following matrix to reduced row echelon form. At each step, notate the exact row operation you are using.

$$\begin{bmatrix} 2 & -1 & -1 \\ -4 & 2 & 2 \\ -1 & \frac{1}{2} & 1 \end{bmatrix}$$

$$\begin{array}{l} R_2 + 2R_1 \\ R_3 + \frac{1}{2}R_1 \end{array} \longrightarrow \begin{bmatrix} 2 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 2 & -1 & -1 \\ 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{l} \frac{1}{2}R_1 \\ 2R_2 \end{array} \longrightarrow \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 + \frac{1}{2}R_2} \begin{bmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

2. (20 points) The following matrix is the augmented matrix of a system of linear equations in the variables $x_1, x_2, x_3, x_4,$ and x_5 .

$$M = \left[\begin{array}{ccccc|c} 1 & 2 & -4 & 4 & 3 & 1 \\ 0 & 3 & -6 & 8 & 1 & -2 \\ 1 & 1 & -2 & 1 & 3 & 2 \\ 1 & -1 & 2 & -4 & 2 & 3 \end{array} \right]$$

Mercifully, your instructor has decided to give you its reduced row echelon form:

$$R = \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & -2 & 0 & 3 & 2 \\ 0 & 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

- (a) What are the basic variables?

$$x_1, x_2, x_4$$

- (b) What are the free variables?

$$x_3, x_5$$

- (c) What are the pivot columns of M ? (Write them as column vectors.)

$$\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \\ 1 \\ -4 \end{bmatrix}$$

- (d) What is the rank of M ?

$$3$$

3. (15 points) Find all of the solutions to the following system of linear equations.

$$x_1 + x_2 - 3x_3 = -2$$

$$x_1 + x_2 + x_3 = 2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -3 & -2 \\ 1 & 1 & 1 & 2 \end{array} \right] \xrightarrow{R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 1 & -3 & -2 \\ 0 & 0 & 4 & 4 \end{array} \right]$$

$$\xrightarrow{\frac{1}{4}R_2} \left[\begin{array}{ccc|c} 1 & 1 & -3 & -2 \\ 0 & 0 & 1 & 1 \end{array} \right] \xrightarrow{R_1 + 3R_2} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

\rightsquigarrow

$$\begin{aligned} x_1 + x_2 &= 1 \\ x_3 &= 1 \end{aligned}$$
 In parametric form:

$$\begin{aligned} x_1 &= 1 - x_2 \\ x_2 &= \text{anything} \\ x_3 &= 1 \end{aligned}$$

4. (25 points) Complete the table to indicate which of the words or phrases "diagonal," "upper triangular," "lower triangular," "in row echelon form," and "in reduced row echelon form" apply to each of the following matrices. Write "Y" if the word or phrase applies to the given matrix and "N" if it does not.

matrix	diagonal	upper Δ	lower Δ	row echelon form	reduced r.e.f.
$\begin{bmatrix} 1 & 3 & 4 \\ 2 & 0 & 0 \end{bmatrix}$	N	N	N	N	N
$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	Y	Y	Y	N	N
$\begin{bmatrix} 1 & 3 & 7 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$	N	Y	N	Y	N
$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$	N	Y	N	Y	Y
$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	N	N	N	Y	N