Math 226 Sections 29 and 33 Quiz 4 2/23/18

Name:__________________________

1. Circle the correct form for the partial fraction decomposition of the rational function

$$\frac{5x-7}{(x^4-1)(x^2+3x+2)}$$

$$= \frac{5x-7}{(x^2+1)(x^2-1)(x+1)(x+2)} = \frac{5x-7}{(x^2+1)(x+1)^2(x-1)(x+2)}$$

(a) $\frac{Ax+B}{x^4-1} + \frac{Cx+D}{x^2+3x+2}$

(b) $\frac{Ax+B}{x^4-1} + \frac{C}{x+1} + \frac{D}{x+2}$

(c) $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2} + \frac{E}{(x+1)^3} + \frac{F}{x+2}$

(d) $\frac{Ax+B}{x^2-1} + \frac{Cx+D}{x^2+1} + \frac{E}{x+1} + \frac{F}{x+2}$

(e) $\frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x+1)^2} + \frac{E}{x+2}$

(f) $\frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{x+2} + \frac{Ex+F}{x^2+1}$

2. If an integral is proper, evaluate it. If an integral is improper, either show that it diverges or evaluate it.

(a) $\displaystyle\int_0^2 \frac{1}{(4+x^2)^{3/2}}dx = \int_0^{\frac{\pi}{4}} \frac{1}{(4+4\tan^2\theta)^{3/2}} \cdot 2\sec^2\theta \, d\theta$

$$\left[\begin{array}{l} x = 2\tan\theta \\ dx = 2\sec^2\theta \, d\theta \end{array}\right] = \int_0^{\frac{\pi}{4}} \frac{1}{(4\sec^2\theta)^{3/2}} \cdot 2\sec^2\theta \, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{2\sec^2\theta}{8\sec^3\theta} d\theta = \int_0^{\frac{\pi}{4}} \frac{\cos\theta}{4} d\theta = \frac{\sin\theta}{4}\Big|_0^{\frac{\pi}{4}} = \boxed{\frac{1}{4\sqrt{2}}}$$

(b) $\displaystyle\int_{-\infty}^0 2^x dx = \lim_{t\to-\infty} \int_t^0 2^x dx = \lim_{t\to-\infty} \frac{2^x}{\ln 2}\Big|_t^0$

$$= \lim_{t\to-\infty} \frac{2^0}{\ln 2} - \frac{2^t}{\ln 2} = \boxed{\frac{1}{\ln 2}}$$

Bonus. Draw a picture of a frustum.

See the video for section 8.2.