Math 226 Sections 29 and 33 Quiz 1 (make up)

Name:_____

1. Find the inverse of $f(x) = 5e^{2x-1}$.

$$y = 5e^{2x-1}$$

$$\frac{y}{5} = e^{2x-1}$$

$$\ln\left(\frac{y}{5}\right) = 2x - 1$$

$$2x = \ln\left(\frac{y}{5}\right) + 1$$

$$x = \frac{\ln\left(\frac{y}{5}\right) + 1}{2}$$

$$y = \frac{\ln\left(\frac{x}{5}\right) + 1}{2}$$

$$\boxed{f^{-1}(x) = \frac{\ln\left(\frac{x}{5}\right) + 1}{2}}$$

2. Evaluate the integral $\int \left(1 + \frac{1}{x}\right)^2 dx$.

$$\int \left(1 + \frac{1}{x}\right)^2 dx = \int 1 + \frac{2}{x} + \frac{1}{x^2}\, dx$$

$$= \boxed{x + 2\ln|x| - \frac{1}{x} + C}$$

3. Find the derivative of $y = x^{\frac{1}{x}}$.

$$\ln y = \ln\left(x^{\frac{1}{x}}\right)$$

$$\ln y = \frac{1}{x}\ln x$$

$$\frac{y'}{y} = -\frac{1}{x^2}\ln x + \frac{1}{x^2}$$

$$y' = \frac{1}{x^2}(1 - \ln x)\, y$$

$$\boxed{y' = \frac{1}{x^2}(1 - \ln x)\, x^{\frac{1}{x}}}$$