Math 227 Sections 7 and 10 Quiz 8 11/8/17

Name: Solutions

1. If a statement is true, circle **T**. If a statement is false, circle **F**.

T **F** If the sequence $\{a_n\}$ converges, then the series $\sum_{n=1}^{\infty} a_n$ converges.

T **F** If the series $\sum_{n=1}^{\infty} a_n$ converges, then the sequence $\{a_n\}$ converges.

T **F** If the sequence $\{s_n\}$ of partial sums converges, then the series $\sum_{n=1}^{\infty} a_n$ converges.

T **F** Every geometric series converges.

T **F** If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge, then so does $\sum_{n=500}^{\infty} (a_n - 2b_n)$

2. Find the second partial sum s_2 of the series $\sum_{n=1}^{\infty} \frac{n}{(n^2+1)^2}$.

$$s_2 = a_1 + a_2 = \frac{1}{(1^2+1)^2} + \frac{2}{(2^2+1)^2} = \frac{1}{4} + \frac{2}{25} = \frac{33}{100}$$

3. Use your answer above and the Remainder Estimate for the Integral Test to find an upper bound and a lower bound for the convergent series $\sum_{n=1}^{\infty} \frac{n}{(n^2+1)^2}$. Simplify all of your answers completely.

$$\int_n^{\infty} \frac{x}{(x^2+1)^2}\,dx = \lim_{t\to\infty} \int_n^{t} \frac{x}{(x^2+1)^2}\,dx \qquad u = x^2+1 \quad du = 2x\,dx$$

$$= \lim_{t\to\infty} \int_{n^2+1}^{t^2+1} \frac{1}{2u^2}\,du = \lim_{t\to\infty} -\frac{1}{2u}\Big|_{n^2+1}^{t^2+1} = \lim_{t\to\infty} -\frac{1}{2(t^2+1)} + \frac{1}{2(n^2+1)}$$

$$= \frac{1}{2(n^2+1)}$$

$$\int_3^{\infty} f(x)\,dx \le s - s_2 \le \int_2^{\infty} f(x)\,dx$$

$$\frac{1}{20} = \frac{1}{3^2+1} \le s - \frac{33}{100} \le \frac{1}{2^2+1} = \frac{1}{10}$$

$$\frac{38}{100} \le s \le \frac{43}{100}$$