Math 226 Sections 7 and 10 Quiz 5 10/4/17

Name:____________________

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Solutions

Evaluate each integral.

1. $\int \frac{x^4+9x^2+5x+18}{x^3+9x}dx = \int x + \frac{5x+18}{x^3+9x}\,dx = \int x + \frac{5x+18}{x(x^2+9)}\,dx$

$$\frac{5x+18}{x(x^2+9)} = \frac{A}{x} + \frac{Bx+C}{x^2+9}$$

$$5x+18 = A(x^2+9) + (Bx+C)x$$

$$5x+18 = (A+B)x^2 + Cx + 9A$$

$$\begin{Bmatrix} 0 = A+B \\ 5 = C \\ 18 = 9A \end{Bmatrix}$$

$$\Downarrow$$

$$\begin{Bmatrix} A = 2 \\ B = -2 \\ C = 5 \end{Bmatrix}$$

$$\int \frac{2x}{x^2+9}dx = \int \frac{1}{u}du$$

$$u = x^2+9$$

$$du = 2x\,dx$$

$$\int \frac{1}{u}du = \ln|u| + C$$

$$= \ln|x^2+9| + C$$

$$= \int x + \frac{2}{x} + \frac{-2x+5}{x^2+9}\,dx$$

$$= \frac{x^2}{2} + 2\ln|x| - \int \frac{2x}{x^2+9}dx + \int \frac{5}{x^2+9}dx$$

$$= \frac{x^2}{2} + 2\ln|x| - \int \frac{2x}{x^2+9}dx + \frac{5}{3}\arctan\left(\frac{x}{3}\right)$$

$$= \boxed{\frac{x^2}{2} + 2\ln|x| - \ln(x^2+9) + \frac{5}{3}\arctan\left(\frac{x}{3}\right) + C}$$

2. $\int_0^{\pi} \sec^2(\theta)d\theta$

This is improper because $\sec^2(\theta)$ has a vertical asymptote at $\theta = \frac{\pi}{2}$.

$$\int_0^{\frac{\pi}{2}} \sec^2(\theta)d\theta = \lim_{t\to\frac{\pi}{2}^-} \int_0^t \sec^2(\theta)d\theta = \lim_{t\to\frac{\pi}{2}^-} \tan(\theta)\Big|_0^t$$

$$= \lim_{t\to\frac{\pi}{2}^-} \tan(t) - \tan(0) = \infty$$

This implies that $\boxed{\int_0^{\pi} \sec^2(\theta)d\theta \text{ diverges.}}$

3. $\int_1^\infty \frac{1}{x^2+x}dx = \int_1^\infty \frac{1}{x(x+1)}dx = \int_1^\infty \frac{A}{x} + \frac{B}{x+1}\,dx = \int_1^\infty \frac{1}{x} + \frac{-1}{x+1}\,dx$

$$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$

$$1 = A(x+1) + Bx$$

$$1 = (A+B)x + A$$

$$\begin{Bmatrix} 0 = A+B \\ 1 = A \end{Bmatrix}$$

$$\Downarrow$$

$$\begin{Bmatrix} A = 1 \\ B = -1 \end{Bmatrix}$$

$$= \lim_{t\to\infty} \int_1^t \frac{1}{x} - \frac{1}{x+1}\,dx$$

$$= \lim_{t\to\infty} \left(\ln|x| - \ln|x+1|\right)\Big|_1^t$$

$$= \lim_{t\to\infty} \ln|t| - \ln|t+1| - \ln 1 + \ln 2$$

$$= \lim_{t\to\infty} \ln\left(\frac{2t}{t+1}\right) = \ln\left(\lim_{t\to\infty}\frac{2t}{t+1}\right)$$

$$= \boxed{\ln 2}$$