Math 226 Sections 7 and 10 Quiz 2 Solutions 9/6/17

Name:______________________

1. Find $f^{-1}(x)$ if $f(x) = (\ln(x^5))^3$.

$$y = (\ln(x^5))^3$$

$$\sqrt[3]{y} = \ln(x^5)$$

$$e^{\sqrt[3]{y}} = x^5$$

$$\sqrt[5]{e^{\sqrt[3]{y}}} = x$$

$$\boxed{f^{-1}(x) = \sqrt[5]{e^{\sqrt[3]{x}}}}$$

2. Differentiate $y = x^{2x}$.

$$\ln y = \ln(x^{2x})$$

$$\ln y = 2x \ln x$$

$$\frac{y'}{y} = 2 \ln x + \frac{2x}{x} = 2 \ln x + 2$$

$$\boxed{y' = (2 \ln x + 2) x^{2x}}$$

3. 11 tribbles are released on the Starship Enterprise. Tribbles reproduce at a rate proportional to their population. After 12 hours there are 121 tribbles aboard the Enterpirse.

(a) Find an expression for the number of tribbles after t hours.

$$P(t) = P(0)e^{kt} = 11e^{kt}$$

$$P(12) = 11e^{k\cdot 12} = 121$$

$$e^{k\cdot 12} = 11$$

$$k\cdot 12 = \ln(11)$$

$$\boxed{k = \frac{\ln 11}{12}}$$

$$\boxed{P(t) = 11e^{kt}}$$

(b) How many tribbles will there be after 18 hours?

$$P(18) = \boxed{11e^{k\cdot 18}}$$

(c) When will the population reach 1,770,000 tribbles?

$$11e^{t\cdot k} = 1{,}770{,}000$$

$$e^{t\cdot k} = \frac{1{,}770{,}000}{11}$$

$$t\cdot k = \ln\left(\frac{1{,}770{,}000}{11}\right)$$

$$t = \boxed{\frac{\ln\left(\frac{1{,}770{,}000}{11}\right)}{k} \text{ hours}}$$