Sec 29 Lagrange multipliers

$f(x,y)$, create $g(x,y)$

Put it together to ge $F(x,y,\lambda)$

$$F(x,y,\lambda) = \underbrace{f(x,y)}_{\substack{\text{max or} \\ \text{min} \\ \text{fcn}}} + \lambda \, \underbrace{g(x,y)}_{\substack{\text{constraint} \\ \text{fcn.} \\ \text{you create} \\ \text{from given}}}$$

Lagrange multiplier

Generally, the problem is stated so that the answer (x,y) will be the one you need to be max or min.

Still, if asked to verify that the (x_0,y_0) is indeed to soln, , do this:

Claim $f(x_0,y_0)$ = max ~~or min~~ value of f

Check by looking at values near (x_0,y_0)

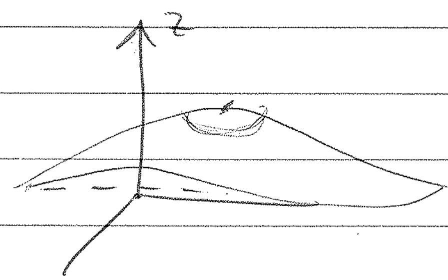
Suppose $x = 400$, $y = 200$

Check at $\underbrace{x = 401, y = 199}_{\text{Based on } g(x)}$

Then, if $f(\overset{400,200}{x_0, y_0})$ is the max,

$f(401,199)$ will be less than $f(400,200)$

$z = f(x,y)$



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2. a) $f(x,y) = 2xy$ maximize

Subject to $x+y=12$

Turn into constraint fcn: $x+y-12=0$

Call this $g(x,y)$

Create $F(x,y,\lambda) = \underbrace{2xy}_{f(x,y)} + \lambda \underbrace{(x+y-12)}_{g(x,y)}$

$F(x,y,\lambda) = 2xy + \lambda x + \lambda y - 12\lambda$

$$\left.\begin{array}{ll} (1) & F_x = 2y + \lambda = 0 \\ (2) & F_y = 2x + \lambda = 0 \\ (3) & F_\lambda = x + y - 12 = 0 \end{array}\right\} \text{Seek crit. pts.}$$

(1) − (2): $2y - 2x = 0 \rightarrow x = y$

(3) $y + y - 12 = 0$

$2y = 12$

$\boxed{y = 6, \quad x = 6}$ $\rightarrow$ $f(x,y) = 2xy$

$f(6,6) = 72$

2b) $f(x,y) = x^2 + 3y^2$

$g(x,y) = x - y + 1 = 0 \rightarrow g(x,y)$

$F(x, y, \lambda) = x^2 + 3y^2 + \lambda(x - y + 1)$

$= x^2 + 3y^2 + \lambda x - \lambda y + \lambda$

① $F_x = 2x + \lambda = 0$

② $F_y = 6y - \lambda = 0$

③ $F_\lambda = x - y + 1 = 0$

① + ② $= 2x + 6y = 0$

Rewrite one in terms of other

$2x = -6y$

into

$x = -3y$

③ $-3y - y + 1 = 0 \rightarrow \boxed{y = 1/4}$

Into constraint $x - y + 1 = 0$

$x - 1/4 + 1 = 0$

$x = -3/4$

$(-3/4, 1/4)$ into $f(x,y) = x^2 + 3y^2$

$f(-3/4, 1/4) = \frac{9}{16} + 3\left(\frac{1}{16}\right) = \frac{12}{16} = \boxed{\frac{3}{4}}$

~~Max~~ $\boxed{\text{Min} = 3/4 \text{ at } (-3/4, 1/4)}$

#4 $C(x,y) = x^2 + 3y^2 + 2000$

$g(x,y)$
$0 = x + y - 1200$ ← $\boxed{x + y = 1200 \text{ necklaces}}$

$$F(x,y,\lambda) = x^2 + 3y^2 + 2000$$
$$+ \lambda(x + y - 1200)$$

$$F(x,y,\lambda) = x^2 + 3y^2 + 2000$$
$$+ \lambda x + \lambda y - 1200\lambda$$

1. $F_x = 2x + \lambda = 0$
2. $F_y = 6y + \lambda = 0$
3. $F_\lambda = x + y - 1200 = 0$

(1) − (2) $= 2x - 6y = 0$

$$\boxed{x = 3y}$$

Into (3)

$$3y + y - 1200 = 0$$
$$4y = 1200$$
$$y = 300$$

Into constraint $x + y = 1200$

$$x + 300 = 1200$$
$$x = 900 \text{ plant A}$$
$$y = 300 \text{ plant B}$$

$C(x,y) = x^2 + 3y^2 + 2000$

$P(x) = R(x) - C(x)$

$P(x,y) = R(x,y) - C(x,y)$

~~P(900,300) = 1000(1000)~~

$$P = \underbrace{1000}_{p}\underbrace{(1200)}_{q} - \underbrace{(900^2 + 3(300^2) + 2000)}_{C(900,300)}$$

$= 1,200,000 - (810,000 + 270,000 + 2000)$

$$\begin{array}{r} 1080 \\ 2 \\ \hline 1082 \end{array}$$

$= 1,200,000$

$- 1,082,000$

$$\begin{array}{r} 1200000 \\ 1082000 \end{array} \qquad \begin{array}{r} 1200000 \\ 1082000 \\ \hline 118000 \end{array}$$