Sec 23 - Optimization HW

1. $x+y=150$, $x^2y = x^2(150-x) = 150x^2 - x^3$

$y = 150 - x$ $\quad f(x) = 150x^2 - x^3$

$x = 100$, $y = 50$ $\quad f'(x) = 300x - 3x^2 = 3x(100-x) = 0$

$\cancel{x=0}$, $x = 100$

What is the maximum product x^2y?

$100^2 \cdot 50 = 50,000$

2. $\boxed{lw = A}$ w, l Area $lw = 3600 \rightarrow l = \frac{3600}{w}$

Perimeter $2(l+w) = 2\left(\frac{3600}{w} + w\right) = \frac{7200}{w} + 2w$

$P(w) = \frac{7200}{w} + 2w$, $P'(w) = -7200w^{-2} + 2 = 0$

$-\frac{7200}{w^2} = -2 \rightarrow w^2 = 3600$

$w = 60\,m$

$A = lw = l(60) = 3600\,m^2$

$l = \frac{3600}{60} = 60\,m$

4. $p(x) = 4 - \frac{x}{12}$, x thousand items

$R(x) = x \cdot p(x) = x\left(4 - \frac{x}{12}\right) = 4x - \frac{x^2}{12}$

$R'(x) = 4 - \frac{2x}{12} = 4 - \frac{x}{6} = 0 \rightarrow 24 - x = 0$, $x = 24$

max rev. $R(24) = 24\left(4 - \frac{24}{12}\right) = 24(4-2) = 48$ $\boxed{\$48,000}$

5. $P'(x) = R'(x) - C'(x)$, since $P(x) = R(x) - C(x)$

Also, $R'(x) - C'(x) = 0$, when $R'(x) = C'(x)$

$P'(x) = 70 - x - .1x^2 - 4x - 10 = 60 - 5x - .1x^2 = 0$

that is, $600 - 50x - x^2 = (60 + x)(10 - x) = 0$

$\cancel{x = -60}$, $x = 10$

6. Given $p(x)$, $C(x)$, and $R(x)$ here: $p(x) = 10 - \frac{x}{400}$

$C(x) = 400 + 2x + .05x^2$, $R(x) = x \cdot p(x) = x\left(10 - \frac{x}{400}\right)$

Find prod. level x to maximize profit.

$R(x) = 10x - \frac{x^2}{400}$, $\boxed{R'(x)} = 10 - \frac{2x}{400} = 10 - \frac{x}{200}$

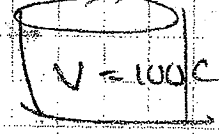
$\boxed{C'(x)} = 2 + .1x$, $P'(x) = R'(x) - C'(x) = 0$

$$P'(x) = 10 - \frac{x}{200} - 2 - .1x =$$

$$= 8 - \frac{x}{200} - \frac{x}{10} = \frac{8 - x - 20x}{200}$$

$$= \frac{1600 - 21x}{200} = 0 \rightarrow 1600 - 21x = 0$$

Prod level: $x = \frac{1600}{21}$ units ≈ 76 units

7. 

$h = ?$
$r = ?$
$V = 100$

Know formula $V = \pi r^2 h$ and that $V = 100$

Substit: $100 = \pi r^2 h$

12. $p = \$30$ $q = 120/\text{mo}$ $p(x) = 30 - 2x$ $q(x) = 120 + 10x$
where x = number of price drops

$R(x) = p(x)q(x) = (30 - 2x)(120 + 10x)$

$R(x) = 3600 + 60x - 20x^2$

$R'(x) = 60 - 40x = 0 \rightarrow x = \frac{3}{2}$ \$2 price drops

$\frac{3}{2} \cdot 2 = \$3$ reduction maximizes revenue

Sell lamp for $\$30 - 3 = \$27$