Sec 11 The rules for derivatives of e^x & $\ln x$ are these:

$$\boxed{\begin{array}{lll} f(x) = e^x & \longrightarrow & f'(x) = e^x \\ f(x) = \ln x & \longrightarrow & f'(x) = \dfrac{1}{x} \end{array}}$$

The proof of the second rule lies in the property:

If $y = x$ then $\ln y = \ln x$

so $y = e^{\ln x}$

and if $y = x$ then $a^y = a^x$.

Ex $f(x) = 3^x$ $\quad f'(x) = 3^x \ln 3$

Ex $f(x) = \log x$ $\quad f'(x) = \dfrac{1}{x \ln 10}$

Ex $f(x) = \ln x$ $\quad f'(x) = \dfrac{1}{x}$

Ex $f(x) = \ln(3x^2 + x - 5)$ $\quad f'(x) = \left(\dfrac{1}{3x^2+x+5}\right)(6x+1)$

~~Ex f(x) =~~

Sec. 4.4 Log + exp differentiation - general

$$\begin{array}{ccc} & f' & \\ e^x & \longrightarrow & e^x \\ \ln x & \longrightarrow & \dfrac{1}{x} \\ a^x & \longrightarrow & a^x \cdot \ln a \\ \log_a x & \longrightarrow & \dfrac{1}{x \ln a} \end{array}$$

~~HW p97 (pp 97 - 98~~

See p. 98 - 99 Problems 4c, d, 5, 6, 13 a, b, c, d, ..., o

These are all good practice

Notice that for all these rules, the chain rule is in play when x is replaced by $f(x)$

General $\log_a(g(x))$ derivatives - notice the $\ln a$ in the denominator

Ex $f(x) = \ln(g(x))$

$$f'(x) = \frac{1}{g(x)} \cdot g'(x)$$

Ex $f(x) = \log_a(g(x))$

$$f'(x) = \frac{1}{g(x) \cdot \ln a} \cdot g'(x)$$

Sec 12 Higher derivatives

$f''(x) \equiv \frac{d^2y}{dx^2} \equiv y''$

$y = \ln(3x)$ $\quad y' = \frac{1}{3x} \cdot 3 = \frac{1}{x}$

$\boxed{y'' = \frac{-1}{x^2}}$ memorize

(general $y = \frac{1}{x^n}$

$\rightarrow y' = \frac{1}{\quad}$

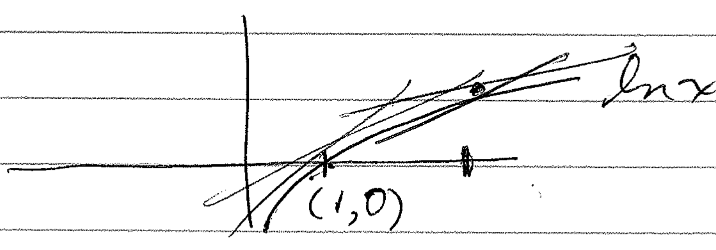
$y = x^{-n}$

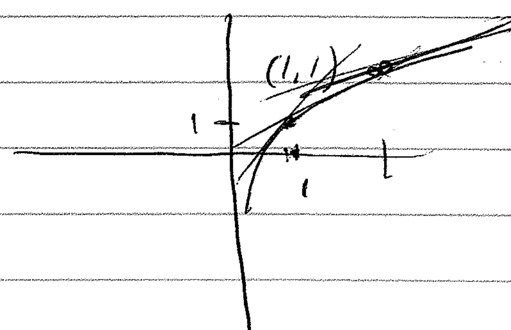
$y' = -nx^{-n-1} = -nx^{-(n+1)} = \frac{-n}{x^{n+1}}$

$y = \ln 3x = \boxed{\ln 3} + \ln x \Rightarrow 1 + \ln x$

2.7

$y' = 0 + \frac{1}{x} = \frac{1}{x}$ same as $\frac{d}{dx}\ln(kx)$



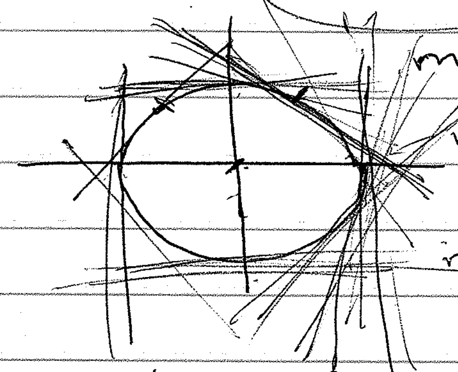


must know x-int of any log fcn

Not surprising $\frac{d}{dx}\ln x = \frac{d}{dx}\ln(kx)$

Sec 13 Implicit Differentiation

Circle $x^2 + y^2 = 1$



Find the slope of the line tangent to the circle at ~~$(1/\sqrt{2})$~~ $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Recipe: $(x_1, y_1) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Employ $y - y_1 = m(x - x_1)$

$$y - y_1 = f'(x_1)(x - x_1)$$

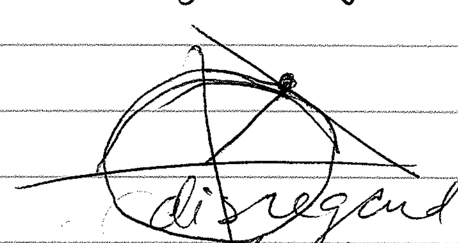
$$y - \frac{1}{\sqrt{2}} = f'(x_1)\left(x - \frac{1}{\sqrt{2}}\right)$$

$m = f'(x) = ??$ But $x^2 + y^2 = 1$ is not an explicit function of ~~x~~ y with respect to x.

Isolate y: $x^2 \cancel{+} y^2 = 1$

$$\sqrt{y^2} = \pm\sqrt{1 - x^2}$$

$$\boxed{y = \sqrt{1 - x^2}} \Rightarrow$$ Use this for derivative $\Rightarrow$ slope



disregard bottom half

Given $x^2 + y^2 = 1$

Find $\frac{dy}{dx}$ implicitly.

1. Treat each term as if it were a fcn. and that each variable is a fcn of x.

2. Differentiate one at a time making sure to multiply by $d\square/dx$, where $\square$ is the variable at hand.

3. Collect like terms in dy/dx + isolate them

4. Solve for dy/dx.

$$\underbrace{x^2}_{\text{fcn of } x \text{ wrt } x} + \underbrace{y^2}_{\text{fcn of } y \text{ wrt } x} = \underbrace{1}_{\text{constant fcn.}}$$

1.

2. $2x \cancel{\frac{dx}{dx}}^{1} + 2y \frac{dy}{dx} = 0$

3. $-2x = +2y \frac{dy}{dx} \longleftrightarrow 2y\frac{dy}{dx} = -2x$

4. $\left.\frac{dy}{dx}\right| = \frac{\cancel{2}x}{-\cancel{2}y} = \left.\frac{-x}{y}\right|_{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)} = \frac{-1/\sqrt{2}}{1/\sqrt{2}}$

$= -1$

$$y = (1-x^2)^{1/2}$$

$$y' = \frac{dy}{dx} = \frac{1}{\cancel{2}}(1-x^2)^{-1/2} \cdot (-\cancel{2}x)$$

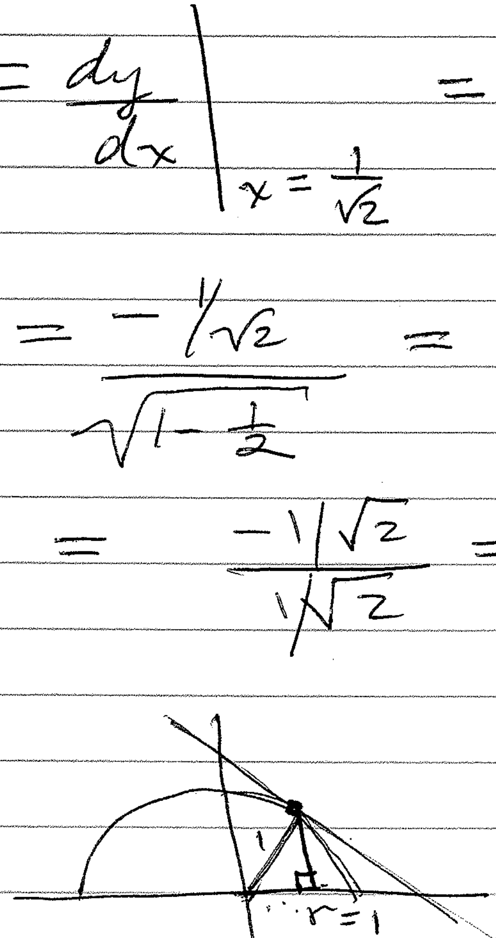
$$m = \frac{-x}{\sqrt{1-x^2}}$$

Evaluate $\overset{y'}{\cancel{m}}$ at $(1/\sqrt{2}, 1/\sqrt{2})$, the notation is

$$\underset{\text{at } x=\frac{1}{\sqrt{2}}}{y'} = \left.\frac{dy}{dx}\right|_{x=\frac{1}{\sqrt{2}}} = \left.\frac{-x}{\sqrt{1-x^2}}\right|_{x=\frac{1}{\sqrt{2}}}$$

$$= \frac{-1/\sqrt{2}}{\sqrt{1-\frac{1}{2}}} = \frac{-1/\sqrt{2}}{\sqrt{1/2}} = \frac{-1/\sqrt{2}}{\sqrt{1}/\sqrt{2}}$$

$$= \frac{-1/\sqrt{2}}{1/\sqrt{2}} = -1$$ slope of tangent to circle at $x = 1/\sqrt{2}$



$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1^2$$

$$y - \frac{1}{\sqrt{2}} = -1\left(x - \frac{1}{\sqrt{2}}\right)$$

Too much work!

Proceed with technique of implicit differentiation using $x^2 + y^2 = 1$

Ex 13.2 $\quad e^{xy} = 5 \quad$ given

find $\frac{dy}{dx}$

Can you isolate y? Heck, no!
Use implicit.

$$\frac{d}{dx} e^{u(x)} = e^{u(x)} \cdot u'(x)$$

$$\text{So } \frac{d}{dx}\left(e^{xy}\right) = \frac{d}{dx}(5)$$

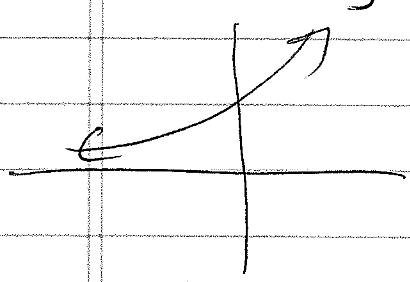
$$e^{xy}\frac{d(xy)}{dx} = 0$$

→ product rule with ID

$$e^{xy}\left(1\,\cancel{\frac{dx}{dx}}\cdot y + x\cdot 1\cdot \frac{dy}{dx}\right) = 0$$

$$e^{xy}\left(y + x\frac{dy}{dx}\right) = 0$$

$$ye^{xy} + xe^{xy}\frac{dy}{dx} = 0$$



$$\frac{dy}{dx} = \frac{-y\cancel{e^{xy}}}{x\cancel{e^{xy}}}$$

$$\boxed{dy/dx = -y/x}$$

your answer will have $x + y$ in it.