Sec. 21 Graphing rational fcns. using FDT, SDT, VA, HA

(#1) $f(x) = \frac{x-1}{x+1}$ $f(0) = \frac{-1}{1} = -1$ **(0, −1) y-int**

$f(x) = 0$ at $x = 1$ (Solve $x - 1 = 0$)   **(1, 0) x-int**

Dom: $x \neq -1$ (write as $(-\infty, -1) \cup (-1, \infty)$)

$$f'(x) = \frac{1(x+1) - (x-1)(1)}{(x+1)^2} = \frac{2}{(x+1)^2} \neq 0$$

Only crit # is $x = -1$, where $f'(x)$ DNE

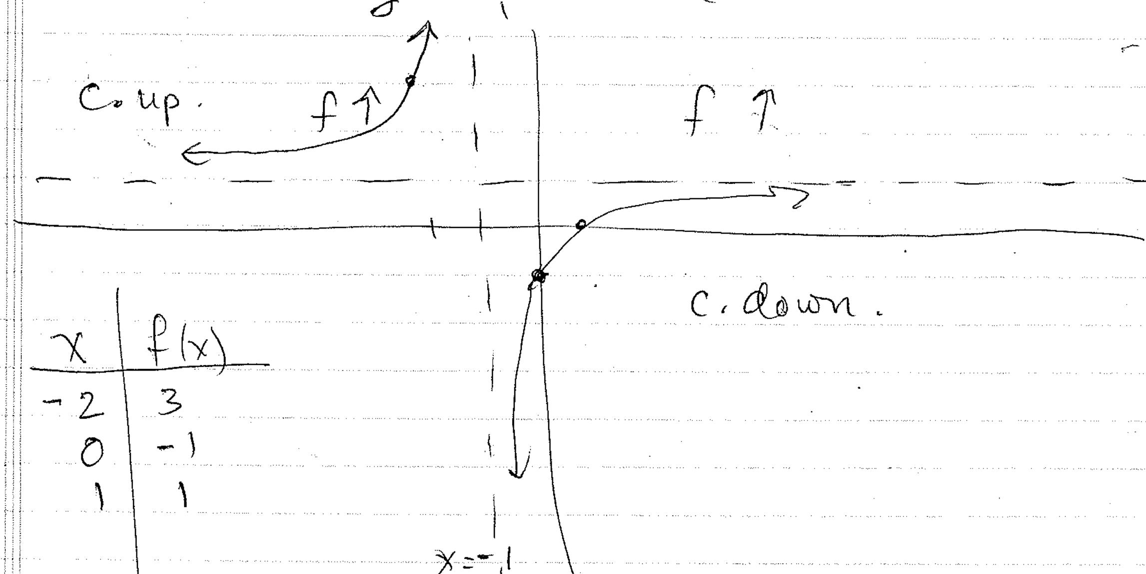
Test $x = -2$: $f'(-2) > 0$
$x = 0$: $f'(0) > 0$
} f increasing on $(-\infty, -1) \cup (-1, \infty)$ i.e. on entire domain, so no local extremes

$$f''(x) = -4(x+1)^{-3} = \frac{-4}{(x+1)^3}$$

Since we can't substitute crit. # $x = -1$ into this, we use SDT on a test value to inspect concavity.

Test $x = -2$: $f''(-2) = -4/(-1)^3 = -4/-1 = 4 > 0$ c. up on $(-\infty, -1)$
$x = 0$: $f''(0) = -4/1^3 = -4 < 0$ c. down on $(-1, \infty)$

Plot some points and then look at concavity.
Also asymptotes! Since deg. $P(x) = \deg Q(x)$, the HA is $y = \frac{1}{1} = 1$ (lead coeff's ratio)



x	f(x)
−2	3
0	−1
1	1

(#2) $f(x) = \dfrac{x^2-3}{x+1}$ $\qquad$ $\deg P(x) > \deg Q(x)$

HA: none $\quad$ ($f(x) \to \pm\infty$ as $x \to \pm\infty$)

$f(0) = -3$, $f(x) = 0$ at $\pm\sqrt{3}$

VA: $x = -1$

$$f'(x) = \frac{2x(x+1) - (x^2-3)(1)}{(x+1)^2} = \frac{x^2+2x+3}{(x+1)^2} = 0 \text{ where?}$$

By Quad. formula: $x = \dfrac{-2 \pm \sqrt{4-12}}{2}$ $\quad$ no real roots so no crit. #s (except $x=-1$)

Test $x=-2$: $f'(-2) = +/+ > 0$
$x = 0$: $f'(0) = +/+ > 0$
} f increasing everywhere so no local extremes

$$f''(x) = \frac{(2x+2)(x+1)^2 - (x^2+2x+3)(2)(x+1)}{(x+1)^4}$$

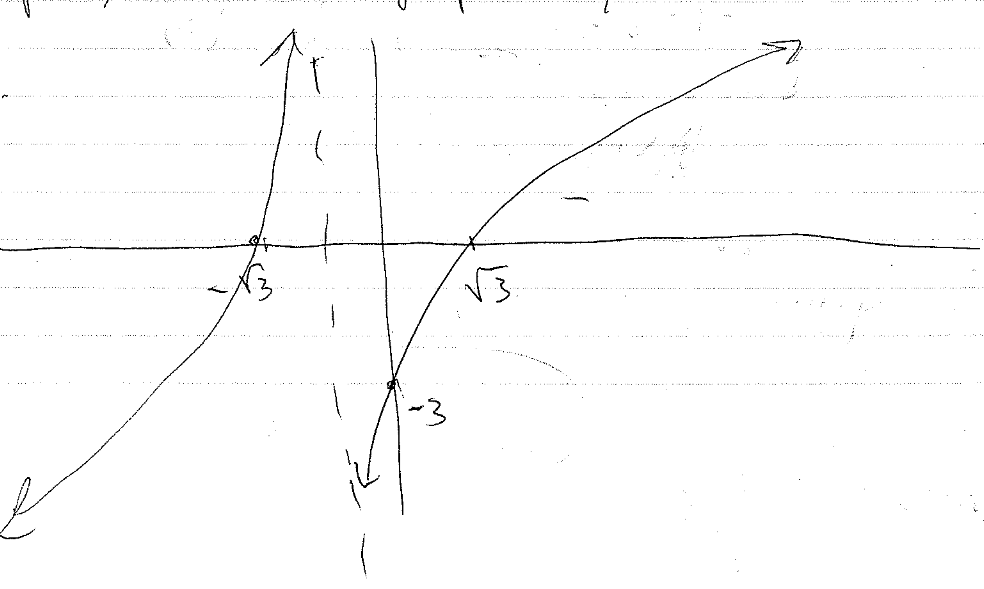
$$= \frac{(x+1)\left[(2x+2)(x+1) - (x^2+2x+3)(2)\right]}{(x+1)^4}$$

$$= \frac{2x^2+4x-4x-2x^2+2-6}{(x+1)^3} = \frac{-4}{(x+1)^3}$$

Test for concavity:
$x=-2$: $f''(-2) > 0$ $\quad$ c. up $(-\infty, -1)$
$x = 0$: $f''(0) < 0$ $\quad$ c. down $(-1, \infty)$

Plot pts, draw asymptotes, look at concavity

x	$f(x)$
0	-3
$-\sqrt{3}$	0
$\sqrt{3}$	0
-2	-1



#3 $f(x) = \frac{x^2}{1+x^2}$ Dom: $\mathbb{R}$

$f(0) = 0$ y-int, x-int

Notice that this fcn is everywhere positive since numerator and denominator are >0 for all x

No VA (denom is valid for all x)

Deg $P(x)$ = Deg $Q(x)$ so HA is $y = 1$ (ratio of lead coeffs)

$$f'(x) = \frac{2x(1+x^2) - x^2(2x)}{(1+x^2)^2} = \frac{+2x}{(1+x^2)^2}$$

$f'(x) = 0$ at $x = 0$, so the intercept is also the crit. #.

Test $x = -1$: $f'(-1) = -/+ < 0$, f decreasing on $(-\infty, 0)$

$x = 1$: $f'(1) = +/+ > 0$, f increasing on $(0, \infty)$

So $x = 0$ has a local min. $(0,0)$

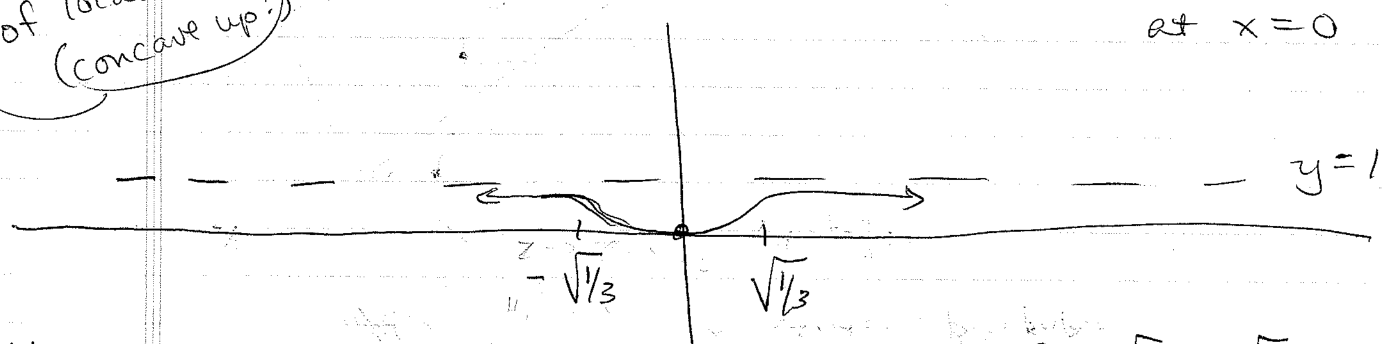
$$f''(x) = \frac{2(1+x^2)^2 - 2x(2)(1+x^2)(2x)}{(1+x^2)^3} = \frac{2(1+x^2) - 8x^2}{(1+x^2)^3}$$

$$f''(x) = \frac{2+2x^2-8x^2}{(1+x^2)^3} = \frac{2-6x^2}{(1+x^2)^3} = \frac{2(1-3x^2)}{(1+x^2)^3}$$

Inflection $f''(x) = 0$ at $x = \pm\sqrt{1/3}$

Check also $f''(0)$ to be sure of local min (concave up?) → $f''(0) = 2(1-0)/(1+0)^3 = +/+ > 0$

so Concave up at $x = 0$



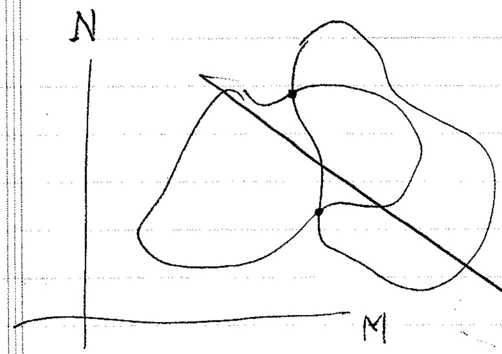
Must be concave down left and right of $-\sqrt{1/3}$, $\sqrt{1/3}$ resp.

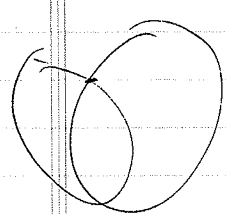
$f''(-1) = -/+ < 0$ and $f''(1) = -/+ < 0$, so this checks.

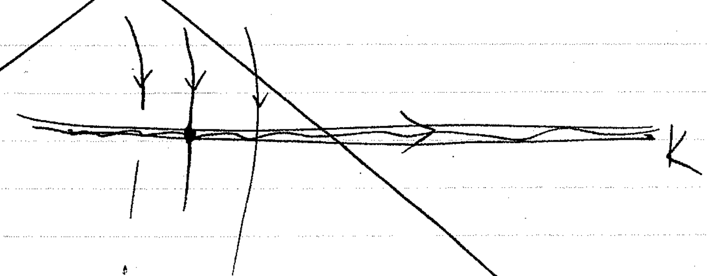
$M \times N \xrightarrow{\Delta \times \Delta} M \times M \times N \times N$

$\alpha = (\alpha_1, \alpha_2)$, $\beta = (\beta_1, \beta_2)$

$[\alpha, \beta] :=$







$\mathfrak{g}_1 \otimes \mathfrak{g}_2$ is Lie if $\mathfrak{g}_1, \mathfrak{g}_2$ has Lie bracket arising from ass. algebra