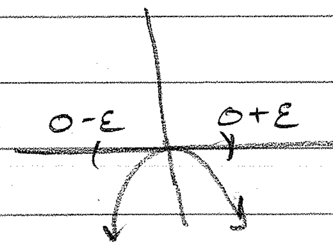
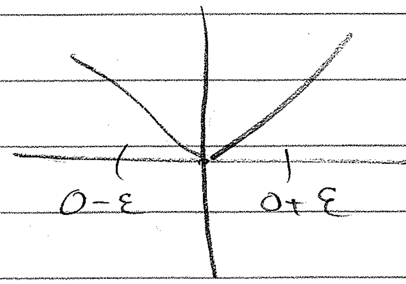
HW #12 Sec 15 - Max / Min / Critical Values

2.a) $f(x) = -x^2$



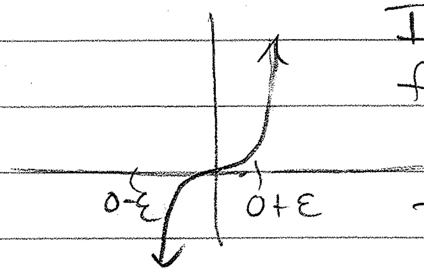
In ε neighborhood of $x=0$, $f(x) \le f(0)$ so $f(0)$ is a local max.

b) $f(x) = |x|$



In ε neighborhood of $x=0$, $f(x) \geq f(0)$ so $f(0)$ is local min

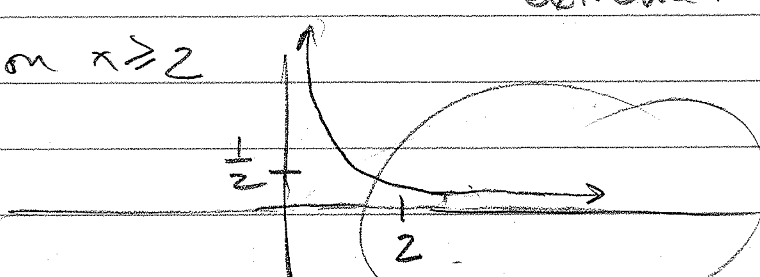
c) $f(x) = x^3$



In ε n'hood of $x=0$, $f(x)$ is neither $\geq f(0)$ nor $\leq f(0)$. Thus, $f(0)$ is not an extreme.

d) $f(x) = \frac{1}{x}$ on $x \geq 2$

Focus on $[2, \infty)$.



There is no ε-neighborhood of $x = 2$, since the domain ends there. So, although $f(2)$ is the greatest value on $[2, 3)$ (say), it does not satisfy the def. of local max.
(You'd need $(2-\varepsilon, 2+\varepsilon)$ to be in domain.)

#4. Find critical points + correspondent critical values.

<u>Note</u> Critical "point" means the x-value where $f'(x)$ either $=0$ or DNE

Critical "value" is the <u>fcn. value</u> there.

c) $f(x) = x^3 - x^2 - 1$ (Note: Dom $f(x)$ is $\mathbb{R}$)

$f'(x) = 3x^2 - 2x = x(3x-2) = 0$ where $x = 0, 2/3$

Both $x=0$ + $2/3$ are in dom $f(x)$ so they are crit pts.

$f(0) = 0^3 - 0^2 - 1 = -1$

$f(2/3) = \left(\frac{2}{3}\right)^3 - \left(\frac{2}{3}\right)^2 - 1 = \frac{8}{27} - \frac{4}{9} - 1 = \frac{8-12-27}{27} = \frac{-31}{27}$

$\boxed{\text{Answers: } (0,-1) + (2/3, -31/27)}$

e) $f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}} \neq 0$, but it DNE at $x=0$

The domain of $f(x)$ is $x \geq 0$, so $x=0$ is a crit pt + $f(0) = 0$

$\boxed{\text{Answer: } (0,0)}$

f) $f(x) = \frac{x^2 - 2x + 1}{x-3}$ <u>Dom</u>: $x \neq 3$

$f'(x) = \frac{(2x-2)(x-3) - (x^2-2x+1)(1)}{(x-3)^2} = \frac{2x^2 - 8x + 6 - x^2 + 2x - 1}{(x-3)^2}$

$f'(x) = \frac{x^2 - 6x + 5}{(x-3)^2} = \frac{(x-5)(x-1)}{(x-3)^2}$ — 3 things to check ↘

1. $f'(x) = 0$ at $(x-5)(x-1) = 0$, $x = 5, 1$

2. $f'(x)$ DNE at $x = 3$

3. Of the values in 1. + 2., which are in domain of $f(x)$? Only the first two, $x = 5, 1$.

$$f(5) = \frac{5^2 - 2(5) + 1}{5 - 3} = \frac{16}{2} = 8$$

$$f(1) = \frac{1^2 - 2(1) + 1}{1 - 3} = \frac{0}{-2} = 0$$

Answers $(5, 8)$ and $(1, 0)$

g) $f(x) = \dfrac{x+1}{x^2+x+1}$ Dom: All x except where

$$x^2 + x + 1 = 0$$

$$\frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)} = \frac{-1 \pm \sqrt{-3}}{2}$$

But the roots of this are

There are no real roots, so dom $= \mathbb{R}$

$$f'(x) = \frac{1(2x+1) - (x+1)(x^2+x+1)}{(x^2+x+1)^2} = \frac{-x^3 - 2x^2}{(x^2+x+1)^2}$$

1. $f'(x) = 0$ where $-x^3 - 2x^2 = 0$.

$$-x^2(x+2) = 0 \Rightarrow x = 0, -2$$

2. $f'(x)$ exists for all x since it's like $f(x)$ in the denominator (cannot = zero)

3. Is $x = 0$ + -2 in dom f? yes! ($x \in \mathbb{R}$)

So $f(0) = 1$ and $f(-2) = -\frac{1}{3}$ are the crit values

$\boxed{\text{Answers: } (0,1) \text{ and } (-2, -\frac{1}{3})}$

h) $f(x) = x + 3x^{2/3}$ Dom: $x \in \mathbb{R}$

$f'(x) = 1 + 2x^{-1/3} = 1 + \frac{2}{x^{1/3}} = \frac{x^{1/3}+2}{x^{1/3}}$

1. $f'(x) = 0$ where $x^{1/3} + 2 = 0$ or $x^{1/3} = -2$

Cube both sides: $(x^{1/3})^3 = (-2)^3$

$\boxed{x = -8}$

2. $f'(x)$ DNE at $x = 0$ since denom is $x^{1/3}$

3. 0, -8 are both in dom f, so they are crit pts.

$f(0) = 0$, $f(-8) = -8 + 3(-8)^{2/3} = -8 + 3(4) = 4$

$\boxed{\text{Answers } (0,0) + (-8,4)}$

i) $f(x) = 4 + (x-1)^{5/3}$ Dom: x is all reals since it is defined for all $x \in \mathbb{R}$ (Why? Ask me in class)

$f'(x) = \frac{5}{3}(x-1)^{2/3} = 0$

at $x = 1$ $f(1) = 4$ $\boxed{(1,4)}$