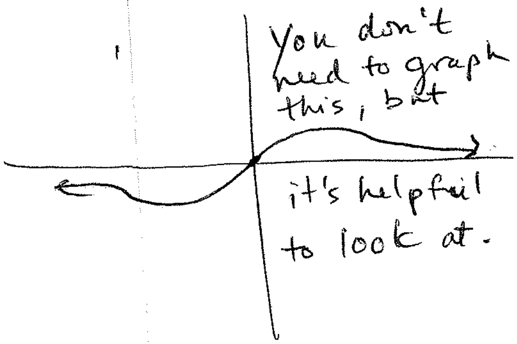
HW 5

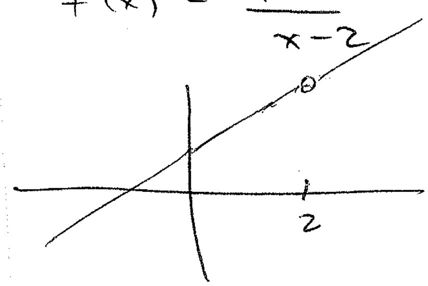
Section 9 – Continuity

1a. $f(x) = x^4 - x^2$ polynomials are cts. on their domain, which is $\mathbb{R}$

b. $f(x) = \dfrac{x}{2x^2+1}$ The domain has no restrictions. The denom is always positive, i.e., it can't = zero or a negative b/c $2x^2+1>0$ for all x. Thus, $f(x)$ is cts. on $\mathbb{R}$. No x where $f(x)$ is discontinuous.

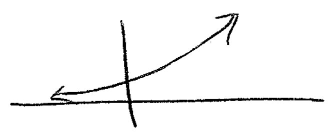


c. $f(x) = \dfrac{x^2-4}{x-2}$ $x=2$ is a value where $f(x)$ is discontinuous.



d. $f(x) = \dfrac{x^2-4x+4}{x^2+x-6} = \dfrac{x^2-4x+4}{(x-2)(x+3)}$ $x \neq 2, -3$

e. $f(x) = 2^x$



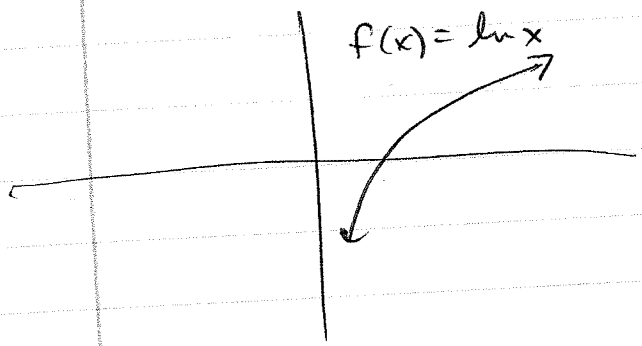
No pts of discontinuity

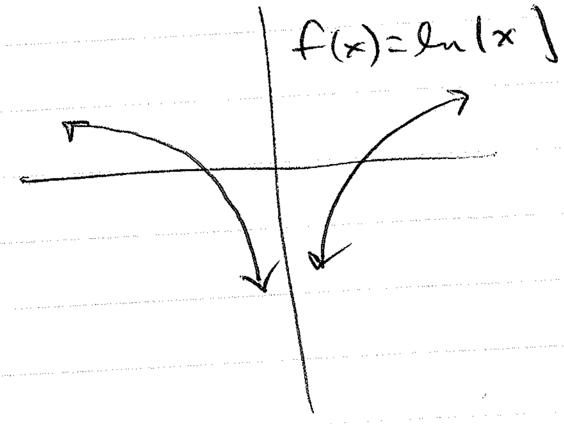
f. $f(x) = |x|$

is cts. everywhere

~~2.~~

but $\ln|x|$ is discontinuous at $x=0$, which is the same as for $\ln x$. The graphs are different, since $\ln|x|$ may take negative values of x. The abs value makes them positive. Comparing the graphs of $\ln x$ and $\ln|x|$ shows why $x=0$ becomes a place where the fcn. is discts. in the second fcn.

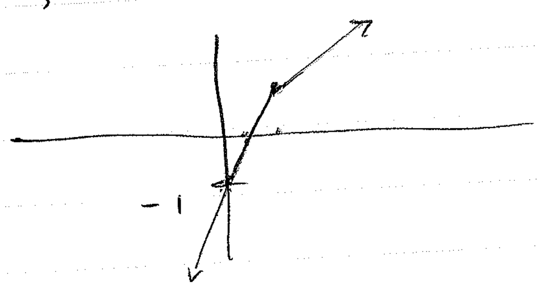




✳ 2.a) $f(x) = \begin{cases} x, & x \le 1 \\ 2x-1, & x > 1 \end{cases}$

The pt. is to check if the pieces connect at $x = 1$

$f(1) = 1, \quad 2(1) - 1 = 1$ so no discontinuity.



b) $f(x) = \begin{cases} 6, & x < -1 \\ x^3+2, & -1 \le x \le 3 \\ 8, & x > 3 \end{cases}$

Look at the fcn. values at the ends of each piece's domain

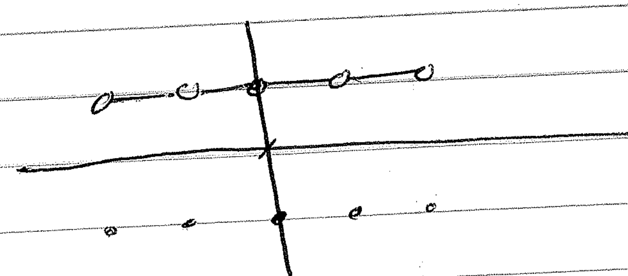
$\left.\begin{array}{l} f(-1) = (-1)^3 + 2 = 1 \ne 6 \\ f(3) = 3^3 + 2 = 11 \ne 8 \end{array}\right\}$ so $x = -1, 3$ have discontinuities

2c) $f(x) = \begin{cases} 4, & x < 0 \\ x, & 0 \le x \le 2 \\ x^2 - 2, & x > 2 \end{cases}$ $\quad f(0) = 0 \ne 4$

$f(2) = 2 = 2^2 - 2$

So $x = 0$ is where the only discontinuity occurs.

d) $f(x) = \begin{cases} -1, & x \in \mathbb{Z} \\ 1, & x \notin \mathbb{Z} \end{cases}$



All the integer values of x are where $f(x)$ is discts.

<u>Question</u>: Is $f(x)$ cts. on $(5,6)$?

<u>Answer</u>: Yes, since $f(x)$ is defined for all values in the interval and $\lim_{x \to c} f(x) = f(c) = 1$ on this or any other interval (a, b)

3. Find c so that $f(x)$ is cts. for all $x \in \mathbb{R}$

$f(x) = \begin{cases} cx - 3, & x < 2 \\ 3 + x + x^2, & x \geq 2 \end{cases}$

Need $cx - 3 = 3 + x + x^2$ at $x = 2$

$2c - 3 = 3 + 2 + 2^2$

$2c = 12$

$c = 6$

$\boxed{c = 6}$

4. $f(x) = \begin{cases} 3x, & x \le 2 \\ ax+b, & 2 < x < 5 \\ -6x, & x \ge 5 \end{cases}$

There are 2 unknowns, a & b, so we write the two equations + solve for a, b.

$3x = ax + b$ where $x = 2$

$-6x = ax + b$ where $x = 5$

$3(2) = 2a + b \longrightarrow 6 = 2a + b$

$-6(5) = 5a + b \longrightarrow -30 = 5a + b$

subtract → to eliminate b

$$\cancel{6 = 2a + b}$$
$$\cancel{-30 = 5a + b}$$
$$\cancel{24 = -3a}$$
$$\cancel{a = 8}$$

$\cancel{6 = 2(8)} + b$

$\cancel{-10} = b$

arithmetic mistake

Take difference

$$6 = 2a + b$$
$$-30 = 5a + b$$
$$36 = -3a$$

$a = -12$

$b = -30 - 5(-12) = -30 + 60 = 30$

$$\boxed{\begin{aligned} -12 &= a \\ 30 &= b \end{aligned}}$$

5. The solution lies in writing an equation that represents the situation. (Modeling)

$1.20/lb when amt < 100 lb
$1.00/lb when amt $\geq$ 100 lb

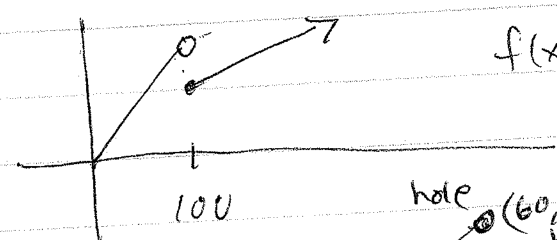
$$f(x) = \begin{cases} 1.20x, & x \leq 100 \\ 1.00x, & x > 100 \end{cases}$$

a) $f(60) = (1.20)(60) = \$72$

$f(200) = (1.00)(200) = \$200$

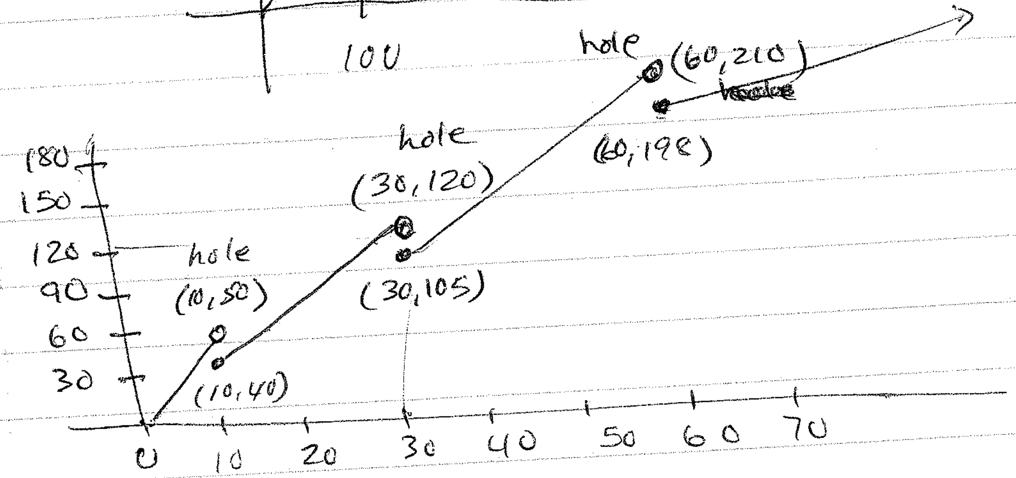
$f(100) = (1.20)(100) = \$120$

b)



$f(x)$ is not cts at $x = 100$

6.



at $x = 10, 30, 60$ the fcn. is discontinuous

7. Calculator useful – shows that as intervals of income change, tax levied does not "jump". There is a smooth transition from one "income bracket" to the next. Just substitute the values at the end of each ~~income~~ bracket & compare.