HW 3 - Compound Interest

1. $P = \$1000 \quad r = .07 \quad t = 10$ yrs

$F = P(1 + \frac{r}{n})^{nt} = 1000\left(1 + \frac{.07}{n}\right)^{n(10)}$

Find F when interest is cmpded

a) Annually $(n=1)$ $\quad F = 1000\left(1 + \frac{.07}{1}\right)^{1\times10}$

$F = 1000(1.07)^{10} = 1000(1.96715)$

$\boxed{F = \$1967.15}$

b) Quartly $(n=4)$ $\quad F = 1000\left(1 + \frac{.07}{4}\right)^{(4)(10)}$

$F = 1000(1.0175)^{40} = \boxed{\$2001.60}$

NOTE: ~~BE PREPARED TO SIMPLIFY~~ $\frac{r}{n}$ W/O USING A CALCULATOR.

For Ex: $\frac{.02}{12} = \frac{2}{1200} = \frac{1}{600} \approx .0167$

$$\begin{array}{r} .00166\ldots \\ 600\overline{)1.0000} \\ \underline{600} \\ 4000 \\ \underline{3600} \\ 400 \end{array}$$

Probably something less clumsy, like

$\left(\frac{.015}{4} = .00375\right)$

c) Monthly $(n=12)$ $\quad F = 1000\left(1 + \frac{.07}{12}\right)^{12\times10}$

$\boxed{F = \$2009.58}$

d) Continuously: new formula $F = Pe^{rt}$

$$F = 1000\, e^{(.07)(10)} = \boxed{\$2013.75}$$

(2.) Effective interest rate: The rate that would actually be paid at 1 year maturity at the quoted annual interest rate. (Book defines this incorrectly - use this description)

If I pay 4% compounded quarterly, I am in effect paying a higher rate, namely:

$$\text{Eff int rate} = \left(1 + \frac{r}{n}\right)^n - 1$$

Notice, $t = 1$, so we don't see it in the exponent.

a) Annual compounding does not enhance the interest. $\left(1 + \frac{.04}{1}\right)^1 - 1 = \boxed{.04}$

b) ~~Quartly~~ Quarterly:

$$\left(1 + \frac{.04}{4}\right)^4 - 1 = (1 + .01)^4 - 1$$

$$= 1.01^4 - 1 = \boxed{0.040604}$$

Be able to do this division w/o calculator.

c) Monthly: $n = 12$, $\text{Eff rate} = \left(1 + \frac{.04}{12}\right)^{12} - 1$

$= \boxed{.040742}$

d) Continuously: The formula changes:

~~lower~~ $\boxed{\text{Eff int rate} = e^{r} - 1}$

You'll have to leave in uncalculated form on a test:

$\boxed{e^{.04} - 1}$

2 part II The question gives an investment principal $P = 90{,}000$ and asks the ~~same~~ questions about eff. int. Since you aren't finding F, there's no change in the answers.

"Principal has no effect on eff int rate; only n does"

3. Find interest earned on $P = \$10{,}000$ at $t = 5\text{ yr}$ at $r = .06$ compounded as follows:

Formula $F - P = P\left(1 + \frac{r}{n}\right)^{nt} - P$

Ann a) $10{,}000\left(1 + \frac{.06}{1}\right)^{5\times1} - 10{,}000 = \boxed{\$3382.26}$

Semi-ann b) $n = 2$, $10{,}000\left(1 + \frac{.06}{2}\right)^{5\times2} - 10{,}000 =$

$\boxed{\$3439.16}$

c) Quarterly $n=4$

$$10{,}000 - 10{,}000\left(1+\frac{.06}{4}\right)^{4\times5} = \boxed{\$3468.55}$$

d) Monthly $n=12$

$$10{,}000 - 10{,}000\left(1+\frac{.06}{12}\right)^{12\times5} = \boxed{\$3488.50}$$

You earned about \$106 more on an investment compounded monthly than on one compounded annually.

* 4. $P = \$3000$ $r = 5\%$ cmpd. continuously
How long (t) to earn \$400 interest?

From $F = Pe^{rt}$, $F = \$3000 + 400 = 3400$
solve the eqn: $\boxed{3400 = 3000e^{.05t}}$

First divide by 3000, then take ln of each side:

$$\frac{34\cancel{00}}{30\cancel{00}} = e^{.05t}$$

$$\frac{17}{15} = e^{.05t} \rightarrow \ln\frac{17}{15} = \ln e^{.05t}$$

$$\rightarrow \ln\frac{17}{15} = .05t \cancel{\ln e}^{\,1}$$

$$\boxed{\frac{\ln 17/15}{.05} = t} \quad \boxed{t \approx 2.503 \text{ yr}}$$

5. Doubling time is found by letting $F = 2P$

a) $2\cancel{P} = \cancel{P}(1 + .035)^{.035t}$

$$\boxed{2 = 1.035^{.035t}}$$

At this pt. you would take the log of each side to a handy base, i.e., what you have on the calculator or in a table. Since we aren't using calculators on the test, you could use any handy base (e.g. base 2) and leave your answer unsimplified, like we did on the quiz.

$$2 = 1.035^{.035t} \rightarrow \log_2 2 = \log_2 1.035^{.035t}$$

$$\longrightarrow \quad 1 = \log_2 1.035^{.035t}$$

$$\longrightarrow \quad \cancel{1 = 0 \log} \quad 1 = .035t \log_2 1.035$$

$$\longrightarrow \boxed{t = \frac{1}{.035 \log_2 1.035}} \approx \boxed{20.149 \text{ yr}}$$

In general, a formula for doubling time is $t = \dfrac{\log 2}{n \log\left(1 + \frac{r}{n}\right)}$ base 10

b) Continuously $\quad 2\cancel{P} = \cancel{P}e^{rt}$

$$2 = e^{rt}$$

$$\ln 2 = rt \cancel{\ln e}^{\,1}$$

$$\ln 2 = rt$$

Formula for doubling at continuous cmpd. $\quad \boxed{t = \frac{\ln 2}{r}}$

$$t = \frac{\ln 2}{.035} \approx 19.804 \text{ yr}$$

* ~~NOTE~~ — Doubling time is unaffected by principal P. It relies only on n and r.

6. Compare investment at 8%, $n = 2$ to one at $7\frac{1}{2}\%$, $n = 12$. $P = \$18{,}000$

$t = 18$ mos $=$

$= 1.5$ yr

* Always convert the months to years.

$$F_{8\%} = 18{,}000\left(1 + \frac{.08}{2}\right)^{2 \times .08}$$

$$F_{7\frac{1}{2}\%} = 18{,}000\left(1 + \frac{.075}{12}\right)^{12 \times .075}$$

The first is better, earning $\sim$ \$111 more interest.

7. W/o a calculator, determine if at $r = 5\%$
$P = \$300$ reaches $F = \$1000$
in 20 yrs. at cont. cmpding.

Solve $F = Pe^{rt}$ for F

$$F = 300\, e^{(.05)(20)}$$

$$F = 300 e^{1} \approx 300(2.718)$$

$$(300)(2.718) < 300(3) < 1000$$

so, $\boxed{t = 20 \text{ is not enough time.}}$

By the way, doubling time at 5% is

$$\boxed{t = \frac{\ln 2}{.05}}$$

Use a calculator to find doubling time t.