Sec 27 p. 219 Selected HW from #1–6

(1b.) $f(x,y) = (x + xy + y)^5$

$f_x = 5(x+xy+y)^4(1+y+0) = 5(x+xy+y)^4(1+y)$

$f_y = 5(x+xy+y)^4(0+x+1) = \cancel{5(x+xy}\ 5(x+xy+y)^4(x+1)$

(1e) $f(x,y) = \ln(3y^8 - 2x)$ $\left[f(x,y) = \ln(u(x,y)) \quad f_x = \frac{1}{u(x,y)} \cdot \frac{\partial u}{\partial x} \right]$

$f_x = \frac{1}{3y^8 - 2x} \cdot (0-2) = \boxed{\frac{-2}{3y^8-2x}}$

$f_y = \frac{1}{3y^8-2x} \cdot (24y^7 - 0) = \boxed{\frac{24y^7}{3y^8-2x}}$

(1f) $f(x,y) = y^3 e^x + x^3 e^y$

$f_x = y^3 e^x + 3x^2 e^y$ [Held y^3 as "coeff" of e^x; then e^y as "coeff" of $\cancel{3}x^3$]

$f_y = 3y^2 e^x + x^3 e^y$ [Held e^x as "coeff" of y^3; then x^3 as "coeff" of e^y]

(1h) $f(x,y) = e^{3x^2} + 2y^3$

$f_x = e^{3x^2} \cdot 6x + 0 = \boxed{6xe^{3x^2}}$

$f_y = 0 + 6y^2 = \boxed{6y^2}$

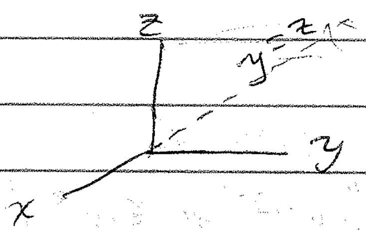
Sec 27

(1i) $z \overset{\text{def}}{\equiv} f(x,y) = y$ This is an interesting example for its simplicity. It's a linear eqn. in 3-space, that is, $ax + by + cz = 0$, where $a = 0$, $b = 1$, $c = -1$. (Recall that $f(x,y) = z$)

$f(x,y)$

Rewrite as $z = y$ for all x
or $0 = y - z$ plane in 3-space (x, c, c)
where (x, y, z) look like ~~(ccc)~~

What is this fcn? It's the plane that cuts through the line $y = z$ ("$y = z$ for all x" is analogous to a horizontal line in 2-space "$y =$ const for all x", or maybe more comparable to "$x = y$" in 2-space, the line through the origin with slope 1).



This line is edge-on look at the plane that comes out of the page.

Now consider f_y: when $f(x,y) = y$, $\boxed{f_y = 1}$

This makes sense, since the slope ~~of the line~~ of the dotted line is 1, and $\dfrac{\partial z}{\partial y} = 1$ at any point (x, y, z) in the plane

Also, $f_x = 0$, since there is no change in z with respect to x. This is a little hard to visualize, but

Same as $f(x,y)$ Same as f_{xx}

(2c) $z = 4x^2e^y$ $\quad \frac{\partial z}{\partial x} = 8xe^y$, $\boxed{\frac{\partial^2 z}{\partial x^2} = 8e^y}$

$\frac{\partial z}{\partial y} = 4x^2e^y$, $\boxed{\frac{\partial^2 z}{\partial y^2} = 4x^2e^y}$

$\frac{\partial^2 z}{\partial x \partial y} = \cancel{8xe^y}$ $8xe^y$, $\boxed{\frac{\partial^2 z}{\partial y \partial x} = \boxed{8xe^y} = \frac{\partial^2 z}{\partial x \partial y}}$

3. $C(x,y) = 16{,}000 + 6x + 20y$

$\frac{\partial C}{\partial x} = 6$, $\frac{\partial C}{\partial y} = 20$

Each ~~\$6~~ change in production of one bicycle ~~produce~~ results in a \$6 change in production cost.

Each change in number of tricycles by one results in a \$20 change in production cost.

4. $M(x,y) = 40x^2 + 30y^2 - 10xy + 30$

$M_y(4,2)$. First find the fcn. M_y, then evaluate it at $(x,y) = (4,2)$

$M_y = 0 + 60y - 10x + 0 = 60y - 10x$, $\boxed{M_x(4,2) = 80}$

$M_x(3,6)$. $M_x = 80x - 10y$; $\boxed{M_x(3,6) = 180}$

if you think of any plane $z = by$ as pages of a book and the x-axis as the spine of the book, you'll see that any change in z with respect to x is zero. For example, since $y = z$ for all x, the points $(0, 1, 1)$ and $(0, 2, 2)$ are in the plane $y = z$; these are on that dotted line. But coming out of the plane of the page, traveling in the x-direction, & we see (x_1, c, c) and (x_2, c, c) are in that plane. $\frac{\partial z}{\partial x}$ could be thought of as

$$\frac{c - c}{x_2 - x_1} = \frac{0}{x_2 - x_1}. \text{ Hence } \boxed{f_x = 0.}$$

(2a) $f(x, y) = 1000 + 5x - 4y^2 - 3xy$

$$f_x = 0 + 5 - 0 - 3y = 5 - 3y$$

$$f_{xx} = 0$$

$$f_y = 0 + 0 - 8y - 3x = -8y - 3x$$

$$f_{yy} = -8$$

$$f_{xy} = \frac{\partial}{\partial y}(5 - 3y) = -3$$

$$\longrightarrow f_{xy} = f_{yx}$$

$$f_{yx} = \frac{\partial}{\partial x}(-8y - 3x) = -3$$