Sec 24 - Elasticity HWD con'd

#8 a) $p(q) = \frac{3}{2}\sqrt{16 - q^2}$

$E(p) = -\frac{p}{q}\frac{dq}{dp}$

We need $q(p)$, not $p(q)$, to get dq/dp.

Solving for q: $p = \frac{3}{2}\sqrt{16-q^2} \longrightarrow \frac{2}{3}p = \sqrt{16-q^2}$

$\longrightarrow * \ \frac{4}{9}p^2 = 16 - q^2 \ *$

You could do an implicit differentiation from here to get dq/dp. We'll still need to isolate p eventually.

By I.D. $\frac{4}{9}\cdot\cancel{2}p = -\cancel{2}q\frac{dq}{dp}$

$\frac{dq}{dp} = -\frac{4}{9}\frac{p}{q}$

Plug into formula $E(p) = -\frac{p}{q}\cdot -\frac{4}{9}\frac{p}{q} = \frac{4p^2}{9q^2}$

Looks like q^2 is all we need. See eqn *

* from this eqn. $\boxed{q^2 = 16 - \frac{4}{9}p^2}$

we get $E(p) = \frac{4}{9} \cdot \frac{p^2}{q^2}$

$$E(p) = \frac{4}{9} \cdot \frac{p^2}{\underbrace{\frac{16}{1} - \frac{4}{9}p^2}_{LCD=9}} = \frac{4}{9} \cdot \frac{9p^2}{16(9) - 4p^2}$$

Simplify: $\boxed{E(p) = \frac{p^2}{36 - p^2}}$ (cancelled 4 top + bottom + 9 top + ~~bottom~~)

b) Is demand elastic $(E > 1)$ or inelastic $(E < 1)$ at price of \$4? What does the answer tell us about how revenue changes with small increase in price at this \$4 level?

$$E(4) = \frac{4^2}{36 - 4^2} = \frac{16}{20} < 1 \text{ (inelastic)}$$

So demand is not pulled (not hurt) by small increases in price, hence revenue will go up if p is increased. Demand remains relatively unchanged so $R = pq$ increases.

(iv)

c) Using the formula $R'(p) = q(1-E)$

find price p so that revenue is maxed.

i.e. Set $R' = 0$ + solve for p.

$$R'(p) = q(1-E)$$

$$= q^{*}\left(1 - \frac{p^2}{36-p^2}\right)$$

* Solved for q from q^2

$$= \sqrt{16 - \frac{4}{9}p^2}\left(\frac{1}{1} - \frac{p^2}{36-p^2}\right) \quad \leftarrow LCD = 36-p^2$$

$$= \left(16 - \frac{4}{9}p^2\right)^{1/2}\left(\frac{36-2p^2}{36-p^2}\right)$$

$$= \left(\frac{16\cdot 9 - 4p^2}{9}\right)^{1/2}\left(\frac{36-2p^2}{36-p^2}\right)$$

factor out 4/9 $\quad = \frac{4}{9}\left(4\cdot 9 - p^2\right)^{1/2}\left(\frac{36-2p^2}{36-p^2}\right)$

$$= \frac{4}{9}\,\frac{\cancel{(36-p^2)^{1/2}}\,(36-2p^2)}{\cancel{(36-p^2)}}$$

reduce by $(36-p^2)^{1/2}$ $\quad = \frac{4}{9}\cdot\frac{36-2p^2}{(36-p^2)^{1/2}} \stackrel{\cdot}{=} 0$

So, $36 - 2p^2 = 0 \rightarrow p^2 = 18$, $p = \$3\sqrt{2}$

Finally, $R(p) = p \cdot q$

$$R(3\sqrt{2}) = (3\sqrt{2})\left(16 - \frac{4}{9}(3\sqrt{2})^2\right)^{1/2}$$

$$= (3\sqrt{2})\left(16 - \frac{4}{\cancel{9}} \cdot \cancel{9} \cdot 2\right)^{1/2}$$

$$= (3\sqrt{2})(16-8)^{1/2}$$

$$= (3\sqrt{2})(\sqrt{8}) = 3\sqrt{2} \cdot 2\sqrt{2}$$

$$= \$11.99 \text{ thousand or } \$11,990$$

The book's answer of \$1,333.33 looks wrong to me. I'll get back to this.